

# Plate Buckling PLII+

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## Basic Documentation – Overview

In addition to the individual program manuals, you will find basic explanations on the operation of the programs on our homepage [www.frilo.com](http://www.frilo.com) in the Campus-download-section.

## Application options

### General scope of application

PLII+ is a calculation software for the analysis of rectangular plates with or without bracing that are exposed to user-defined actions on the edges. In addition to verifying the structural safety (verification of the buckling resistance based on the method of reduced stresses in accordance with EN 1993-1-5, para. 10), you can simply calculate the buckling load factors and buckling coefficients in accordance with the linear buckling theory. The calculation of the buckling field is based on the Finite Element Method (FEM) in this analysis.

All examinations, including the calculation of the ideal buckling values, are performed on the total field and include discrete longitudinal and transverse stiffeners. The effect of the stiffeners on the buckling behaviour is taken into account in all of these examinations. Therefore, verifications for individual or partial fields or an additional verification of the minimum stiffness of stiffeners can be dispensed with. Buckling on a member that behaves like an Eulerian column can also be considered for the entire braced field, if applicable.

PLII+ allows you to calculate any number of load cases within the same item, including the following types of loads:

1. identical trapezoidal line loads applying opposite to each other at the transverse edges
2. identical trapezoidal line loads applying opposite to each other at the longitudinal edges
3. two concentrated loads (e.g. wheel loads from crane operation) applying at the upper edge of the buckling field
4. a freely definable reaction force applying at the bottom edge of the buckling field, e.g. supporting force of a bearing corbel
5. constant circumferential edge shear loading

The loading described in item 3 and 4 allows you to map also variable bending and shear stress distributions in the buckling field by calculating the plate loading on a cantilever or a single- or double-span beam pinned on two sides.

For structures that are not handled in the selected design code (e.g. one or several unsupported buckling field edges), linear buckling loads can be calculated separately for all action components. This can optionally be done in an automatic calculation.

For purposes of illustration, PLII+ displays graphically the following calculation results for each load case. The displayed results depend on the selected design code, i.e. either with plate-type and Eulerian-column-type behaviour (EN 1993-1-5) or with individual edge loads (DIN 18800):

- Stress fields  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$  and  $\sigma_v$
- Buckling mode associated to the lowest buckling load

By defining edge stiffeners to model flanges, PLII+ also allows you to examine stability problems in connection with buckling and lateral torsional buckling causing the deformation of the cross section.

PLII+ delivers the ideal buckling loads with consideration to local buckling in compression-loaded web areas.

The results of such an examination strongly depend on the definition of the individual portions of the stiffeners' stiffnesses and their contribution to the load transfer.

### Special areas of application

The PLII+ software is especially designed for welded beams with very slender webs the buckling safety of which must be verified. The buckling risk may be attributable to the entire supporting behaviour (loading by bending and axial force) and/or locally transferred loading as with craneway beams.

## Design standards

The structural safety verifications in the **PLII+** application are based on EN 1993-1-5, para. 10 (method of reduced stresses) and the regulations of the National Annexes (availability depends on your licence).

- DIN EN 1993-1-5/NA
- ÖNORM B 1993-1-5
- NA to BS EN 1993-1-5

The verification in accordance with DIN 18800 is also implemented. Some calculation options that are available in combination with EN 1993-1-5 are not available with the DIN code, however.

## Definition of the structural system

In **PLII+** you can map exclusively rectangular buckling fields with a constant thickness. The buckling field can be braced with any number of transverse and longitudinal stiffeners. For the calculation of the panel, constraint-free (statically determined) support is assumed. The panel loading is completely determined by the defined edge loads and the associated equilibrium loads generated internally. The boundary conditions for the plate edges (buckling analysis) can be defined for each buckling field edge separately.

## Special notes concerning the system definition

### Buckling stiffeners

In addition to simple buckling stiffeners you can also examine flanges of beams in **PLII+**. This allows you to take more realistic and normally higher buckling values into account in the calculation on the separately considered beam than it would be possible on the simple buckling field.

The stiffeners are arranged orthogonally to the plate edges and can optionally either be distributed over the total length and/or width of the buckling field or placed in particular areas. The system geometry is defined by specifying the front end and rear end coordinates of the stiffeners.

**PLII+** treats stiffeners always discretely, i.e. smearing of stiffeners is neither possible nor necessary. When comparing the results to buckling values of smeared buckling stiffeners, you should note that buckling of individual fields is not included! **PLII+** also considers buckling of the individual fields between the stiffeners in the buckling examination and calculates for each field the smallest eigenvalue with the associated buckling mode.

In the FE modelling, stiffeners are mapped in **PLII+** by three-dimensional beam elements that are connected centrally in the middle of the plate. The influence of possible stiffener eccentricities is considered via effective widths in accordance with the specifications of the selected design code. This method is customary in steel construction.

**PLII+** allows you to take the effects of discrete stiffeners either in the panel and buckling examination or only in the buckling calculation into account. Corresponding control parameters that you set during the definition of the stiffeners provide for this. **PLII+** offers options to take the influence of axial stiffness, torsional stiffness and bending stiffness of stiffeners into account in the calculation of the ideal buckling values.

Buckling stiffeners should be defined as follows:

- flat steel stiffeners - arranged centrally or eccentrically to the central plate area
- angled or T-stiffeners
- trapezoidal stiffeners

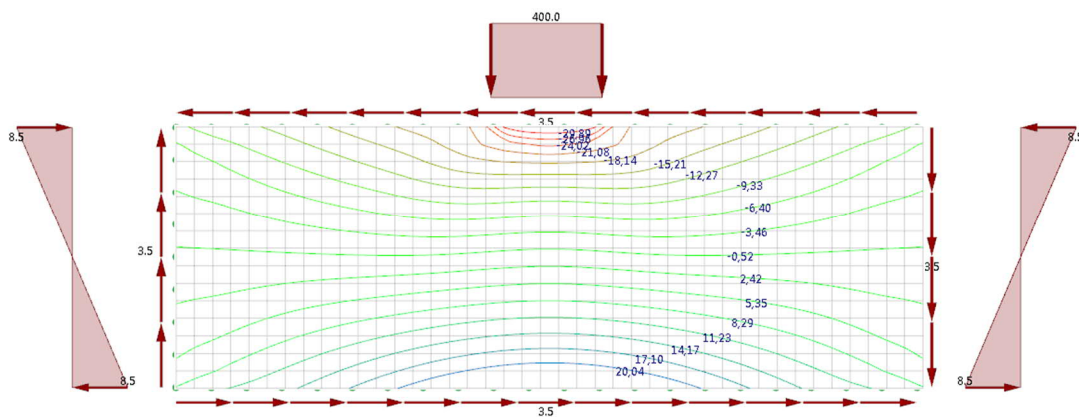
For flat-steel, T-, angled and trapezoidal stiffeners, **PLII+** calculates automatically the effective widths and the resulting effective cross-sectional properties from the stiffener spacing, the plate thickness and the cross-section geometry.

## Load cases and edge loads

### Note concerning the load values

When defining the actions on the edges, keep in mind that **PLII+** expects the specification of loads (distributed or concentrated) not of stresses! If edge loads are derived from previously calculated bending and shear stresses, you must multiply their values with the plate thickness before entering them.

### Notes concerning the load introduction problem



*Illustration:  $\sigma_x$  stress distribution for rectangular plates under concentrated loading.*

As you can see in the illustration, the load introduction problem involves  $\sigma_x$  stresses (bending stresses) that are not constant over the length of the buckling field in addition to  $\sigma_y$  stresses  $\sigma$ . The influence of the bending stresses on the ideal buckling load is only negligible if the  $\alpha$  values are low. When comparing buckling values of long buckling fields to the results obtained in accordance with [10], for instance, you should note that only  $\sigma_y$  stresses have been taken into account in the latter case.

The  $\sigma_x$  stresses are caused by the local load introduction (upper edge of the buckling field), on the one hand, and the global load transfer through bending, on the other hand. In **PLII+**, you can optionally eliminate both causes by calculating only the  $\sigma_y$  stresses by means of FEM and deriving the remaining stress components from the defined edge loading.

## Calculation and analysis

### Special regulations of the National Annexes

You should keep in mind that some National Annexes restrict the application of the verification method. The National Annex for Germany limits the application to plates with pinned supports on all sides, for instance. If your system does not comply with this boundary condition, you can at least determine the buckling load factors, which give a rough indication of the buckling risk if the structure behaves like a plate (unbraced or slightly braced plates with a small length/width ratio). In this case, the real supporting load applies typically above the smallest buckling load due to the overcritical supporting reserves.

All National Annexes recommend limiting the application of the verification method to plate areas under pressure. This recommendation, which should facilitate the engineer's work, is not easy to interpret, however, because some systems involve complex loading scenarios and the phenomenon of shear buckling. PLII+ solves the problem by setting the tensile stress component to zero. Therefore, only compression and tensile stresses are taken into account in the verification equation for buckling failure. In addition, a comparison stress resistance verification of the steel yield strength is performed. All stress components are taken into account in this verification but without the reduction factors for buckling.

### Eulerian column-type behaviour

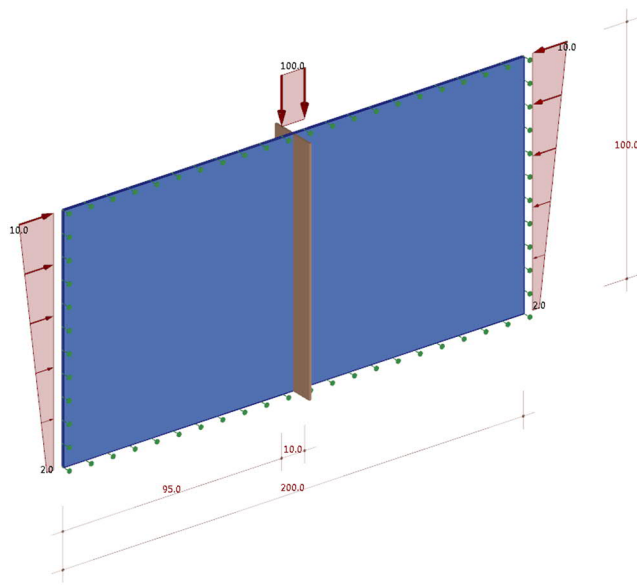
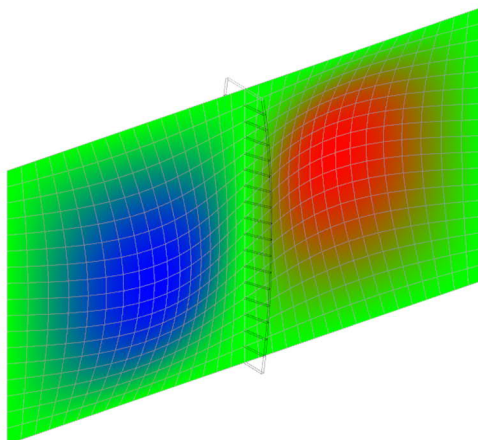
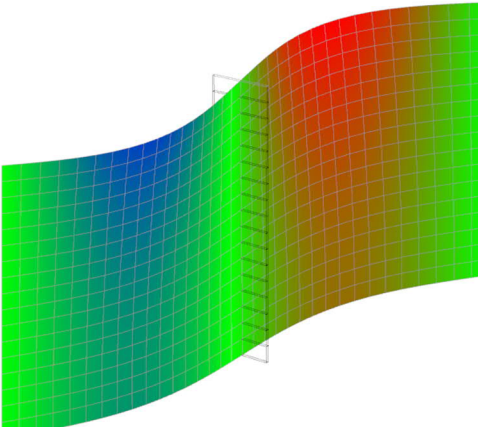
The buckling charts used in EN 1993-1-5 and also DIN 18800 (supporting capacity coefficients depending on ideal component slenderness) constitute an empirical approach, which activates overcritical supporting reserves up to a certain extent. Their validity is limited to plate-type behaviour, which is characterised by a significant load transfer in the transverse direction.

If the load transfer is uniaxial, e. g. because the transverse edges are far away from each other, or a braced buckling field is strongly orthotropic, the buckling curves for bar-type components should be used in the load-bearing strength verification, see EN 1993-1-5, 4.5.1 (5). In reality, most plates show a mixed behaviour of both, plate-type and Eulerian column-type behaviour. Therefore, EN 1993-1-5 (as well as DIN 18800) prescribes interpolation between the reduction factors of both extreme cases.

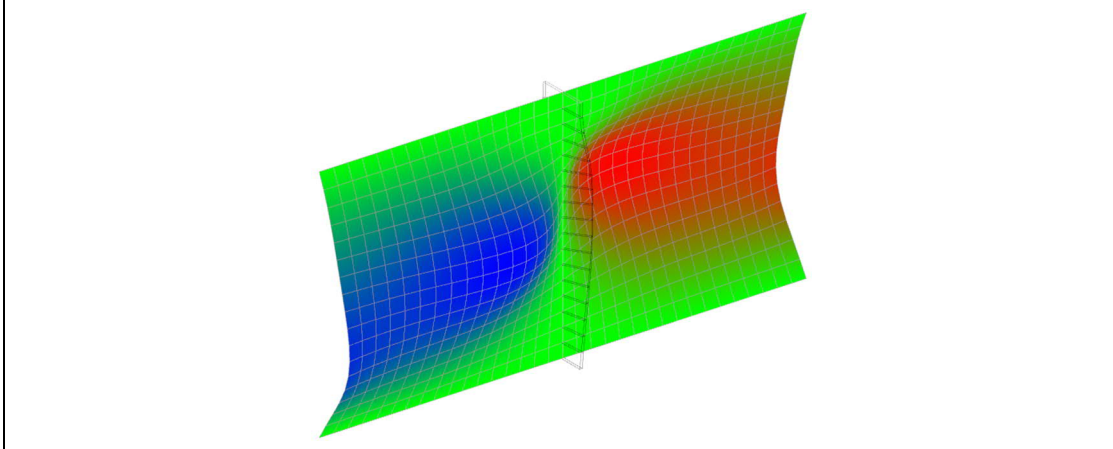
To quantify the interpolation point, the influence of the support of the longitudinal edges on the smallest buckling factor is taken into consideration. For the evaluation of the Eulerian column-type behaviour, another linear buckling analysis is performed, but this time, supporting conditions at the edges in parallel to the loading direction are not considered, see EN 1993-1-5, 4.5.3 (1). The greater the drop of the buckling load in this analysis, the more distinctive the plate-type behaviour (high transverse load transfer). Or the other way round, the smaller the drop of the buckling load, the more distinctive the Eulerian-column behaviour (low transverse load transfer).

For the manual calculation, EN 1993-1-5 offers a simplified calculation of the buckling factor with a set of equations that are based on an Eulerian column. You should note, however, that the typical approaches on the safe side involve an underestimation of the buckling load which will lead to an overestimation of the plate-type behaviour and consequently, to an overestimation of the load-bearing capacity of the plate!

The following table gives an example of how to proceed:

<p>System and loading</p> 
<p>All plate edges have pinned supports → plate-type behaviour (<math>\alpha_{cr,p} = 1.31</math>)</p>

<p>Only the transverse edges have pinned supports → Eulerian column-type behaviour in the x-direction (<math>\alpha_{cr,c,x} = 0.28</math>)</p>


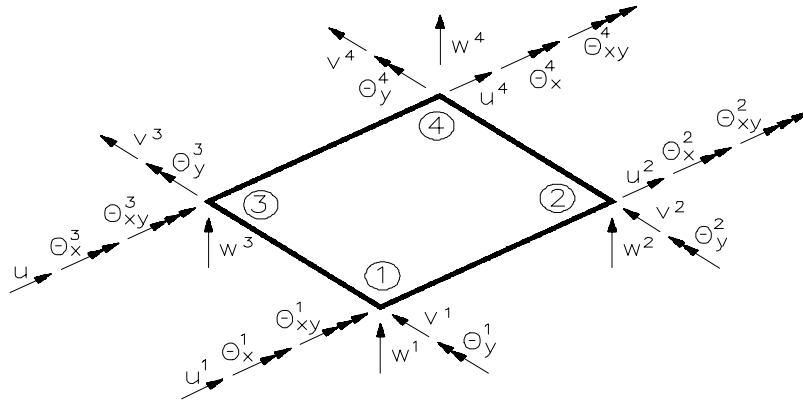
Only the longitudinal edges have pinned supports → Eulerian column-type behaviour in the y-direction  
( $\alpha_{cr,c,x} = 51.2$ )



**Comments:** *The longitudinal direction is decisive for the examination. Even though the transverse stiffener ends on the supported longitudinal edge in each case, the stiffener must also be supported at its ends in order to avoid the underestimation of the buckling load. To exclude local buckling due to load introduction stresses, only those edge loads are taken into account for the Eulerian-column type behaviour that act in the direction of examination. The high buckling load factor in the y-direction emphasizes the dominance of the x-direction in regard to the buckling resistance behaviour.*

## Basis of calculation

### FE element formulation



For numerical examination with the FE method, the considered plate is described by finite, rectangular elements with four nodes each and 24 degrees of freedom all in all. The geometrically non-linear folded plate element is based on the linear, conformal plate element by [Bogner/Fox/Schmit](#) [8] with four nodes and four degrees of freedom each.

Hermite polynomials are used to approximate the node deformation behaviour inside the element. Interelement compatibility perpendicular to the edges (C1 compatibility) is created by one degree of warping freedom at each node. The element is conformal in regard to bending with guaranteed convergence. The convergence behaviour of the plate element lying on the bilinear panel element with four nodes and two degrees of freedom is improved by forcing constant shear distortion. Panel and bending effects are decoupled as typical in linear theory. To create the geometrical stiffness matrices of the plate elements for the eigenvalue examination, all membrane stress components  $\sigma_x$ ,  $\sigma_y$  and  $\tau$  resulting from the various load cases are assumed constant inside an element.

### Eigenvalue examination

The eigenvalue examination in accordance with the linear buckling theory is made iteratively by inverse vector iteration and/or by using the linear subspace method separately for each of the three possible load cases mentioned above. The FE formulation produces the following general matrix eigenvalue problem for the smallest eigenvalue  $\eta_{ki}$ .

$$[K_e + \eta K_g] \Phi = 0$$

$K_e$  is the elastic, linear system stiffness matrix,  $K_g$  the geometrical system stiffness matrix and  $\Phi$  the matrix of the eigenvectors.  $K_g$  is a function of the membrane stress components in the element centre as well as of the geometric element dimensions. Strongly varying membrane stresses, local effects or complex mode shapes, e.g. with plates with several stiffeners, influence the convergence behaviour of the eigenvalue iteration process adversely as much as low membrane stresses do. If the eigenvalue examination via inverse vector iteration does not produce any result, [PLII+](#) launches an eigenvalue examination based on the subspace method.

In the calculation of the ideal buckling loads (bifurcation loads), all membrane stress portions caused by the respective edge loading are taken into account in the geometrical stiffness matrix. If the basic parameters were set accordingly, the influence of the  $\sigma_x$  and  $\tau$  stresses, applying simultaneously, is included in the buckling values in the  $r_y$  load case as in [11].



## Ideal buckling values - unbraced plates

For unbraced rectangular plates with pinned edges, comparative analytical solutions such as those described in [7] and [9] are available.

For rectangular plates with simply supported edges and  $\sigma_x$  edge stresses, which have an edge stress ratio of  $\psi$  and a length/width ratio of  $\alpha = a/b$ , the following table shows the comparison between analytically determined ideal buckling stresses and the values (in italics) calculated by PLII+ on 600 elements in each case.

The following conditions apply: buckling field width  $b = 100$  cm, plate thickness  $t = 1.0$  cm, module of elasticity  $E = 21000$  kN/cm<sup>2</sup>, Poisson's ratio  $\mu = 0.3$

$\psi$	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$	$\alpha = 2.0$
1.0	11.86	7.59	7.59	7.59
	<i>11.86</i>	<i>7.59</i>	<i>8.24</i>	<i>7.59</i>
-1.0	48.40	45.40	45.36	45.36
	<i>48.45</i>	<i>48.47</i>	<i>45.76</i>	<i>45.33</i>

Table: Comparison between buckling values of unbraced plates and  $\sigma_x$  edge stresses

In the table below, ideal buckling stresses for the above mentioned parameters for constant edge shear are compared to the values in italics on the basis of an analytic solution. The values in italics originate from PLII+ calculations on 200 elements in each case.

$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$	$\alpha = 2.0$
48.13	17.73	13.51	12.03
<i>49.72</i>	<i>17.70</i>	<i>13.42</i>	<i>12.43</i>

Table: Comparison of buckling values of unbraced plates to edge shear loads

The comparison shows that the values for  $\sigma_x$  and  $\tau$  edge loads determined in PLII+ correspond roughly to the analytical solutions for unbraced plates.

## Ideal buckling values - braced plates

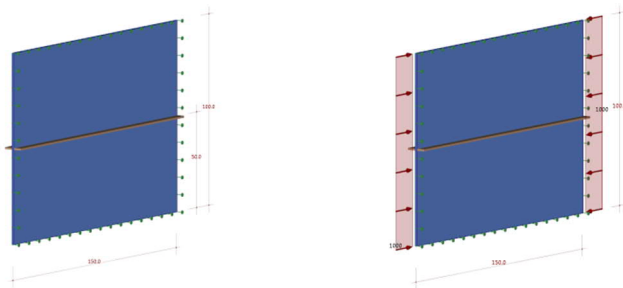
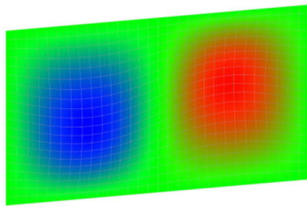
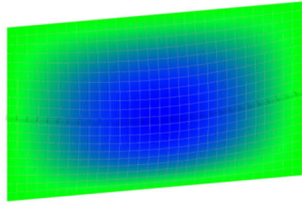
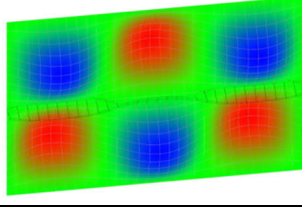
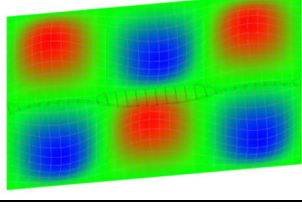
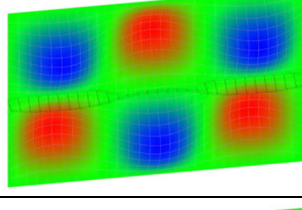
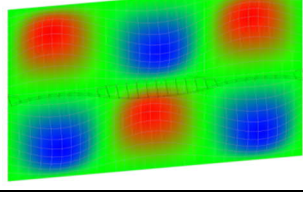


Illustration: System 1 - plate 1500 x 1000 mm with flat steel stiffener 100 x 10,  $\psi = 1.0$

Case	Description	$\sigma_{cr,p,x}$	Buckling mode
i)	Without stiffener	8.24 kN/cm <sup>2</sup>	
ii)	With central stiffener, without consideration of axial stiffness in the panel calculation, without torsional stiffness of the stiffener	23.52 kN/cm <sup>2</sup>	
iii)	With eccentric stiffener, without consideration of axial stiffness in the panel calculation, without torsional stiffness of the stiffener	30.37 kN/cm <sup>2</sup>	
iv)	With eccentric stiffener, with consideration of axial stiffness in the panel calculation, without torsional stiffness of the stiffener	32.43 kN/cm <sup>2</sup>	
v)	With eccentric stiffener, without consideration of axial stiffness in the panel calculation, with torsional stiffness of the stiffener	32.21 kN/cm <sup>2</sup>	
vi)	With eccentric stiffener, with consideration of axial stiffness in the panel calculation, with torsional stiffness of the stiffener	34.37 kN/cm <sup>2</sup>	

## Calculation process

The calculation is divided in three phases.

1. FE panel calculation
2. FE buckling analysis
3. Buckling resistance verification
4. Stability verification of buckling stiffeners

In the first step, the panel stress state is determined under simultaneous action of all edge loads. On the basis of this panel stress, the geometrical stiffness of the plate elements can be calculated subsequently and the eigenvalue problem for plate-type behaviour can be solved. This calculation step is repeated two more times, by ignoring the supporting conditions on two parallel plate edges each time. The buckling load factors for Eulerian column-type behaviour are determined in this step. In the last step, the verification of the buckling resistance is performed in the centre of each finite element.

Possibly applying tension is ignored in this verification. The verification of the plate areas under tension is included in the verification of the cross-sectional resistance to comparison stresses. Subsequently, the stiffening forces of all buckling stiffeners that span over the entire plate under buckling stress are read out from the FE panel calculation and their load-bearing capacity is determined in a second-order analysis taking initial imperfection (and additional forces generated by the development of the tension field generation and deviation force through shear deformation) into account.

*Note: The verification of trapezoidal buckling stiffeners has not been implemented yet.*

## Verifications in accordance with EN 1993-1-5 , para. 10

### Verification concept

The fundamental idea of the method of reduced stresses is the limitation of the elastic stresses in all parts of the cross section to a local supporting buckling stress. The supporting buckling stress corresponds to the yield limit until which the cross section can fully bear the loads reduced by one factor. If the supporting buckling stress is not exceeded in any part of the cross section, it can be classified in class 3. Stress redistribution inside the cross section cannot be taken into account then. The cross-sectional part under the highest loading determines the load-bearing capacity of the entire cross section.

The reduction factor for the yield strength is based on a degree of system slenderness that is determined on the basis of the entire effective stress field. The verification of the individual resistances and the subsequent combination via an interaction criterion can be dispensed with. The verification concept already includes an interaction referring to both, the stress level, based on the comparison stress hypothesis, as well as the reduction factors. For metal plates, the stability of which is not at risk, this criterion is turned into a conventional comparison stress verification.

The verification is as follows: 
$$\frac{\sigma_{v,Ed}}{\sigma_{v,Rd}} \leq 1$$

The variables are:

$\sigma_{v,Ed}$  Design value of the comparison stress of the entire effective stress field

$$\sigma_{v,Ed} = \sqrt{\sigma_{x,Ed}^2 + \sigma_{z,Ed}^2 - \sigma_{x,Ed} \cdot \sigma_{z,Ed} + 3 \cdot \tau_{xz,Ed}^2}$$

$\sigma_{v,Rd}$  Design value of the supporting buckling stress

$$\sigma_{v,Rd} = \rho_p \cdot f_{y,d}$$

$\rho_p$  Reduction factor for plate buckling with consideration of the Eulerian column-type behaviour

The format of the verification is quite simple. The critical challenge is the calculation of the reduction factor. The basis for this calculation is the so-called system slenderness.

The system slenderness is the initial entity for the calculation of the reduction factors for plate buckling and Eulerian column-type behaviour. The referenced system slenderness for plates prone to buckling is determined as follows:

$$\bar{\lambda}_p = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}}$$

The variables are:

$\alpha_{ult,k}$  Factor by which you must multiply the design values of the resistances in order to obtain the required yield strength in the most unfavourable fibre.

$$\alpha_{ult,k} = \frac{1}{\sqrt{\left(\frac{\sigma_{x,Ed}}{f_{y,d}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{f_{y,d}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{f_{y,d}}\right) \cdot \left(\frac{\sigma_{z,Ed}}{f_{y,d}}\right) + 3 \cdot \left(\frac{\tau_{xz,Ed}}{f_{y,d}}\right)^2}} = \frac{f_{y,d}}{\sigma_{v,Ed}} \quad \text{Cf. [1], eq.(10.3)}$$

$\alpha_{cr} = \frac{\sigma_{v,cr}}{\sigma_{v,Ed}}$  Factor by which you must multiply the design values of the resistances in order to obtain the required elastic critical comparison buckling stress in the most unfavourable fibre.

The structure of this term is similar to that for the stability problems lateral buckling and lateral torsional buckling. The dividend represents a limit value not threatening stability whereas the divisor indicates the ideal ultimate load related to the stability problem.

The starting point for the calculation of the system slenderness is the ideal limit buckling stress. If the corresponding buckling values are available, the ideal limit buckling stress is calculated as follows:

$$\sigma_{cr} = k \cdot \frac{\pi^2 \cdot E_k}{12 \cdot (1 - \mu^2)} \cdot \left(\frac{t}{b}\right)^2$$

So, the decisive entity is the buckling value. It corresponds to the effective lengths factors  $\beta$  of the compression member. Buckling values can be taken from reference literature.

If you take a closer look, you can see that the coefficient  $\alpha_{cr}$  is the buckling factor of the total system. If a software for the calculation of the buckling factor on the basis of the Finite Elements Method is available, you can dispense with the roundabout approach via ideal buckling loads.

The system slenderness can be expressed in the following equation:

$$\bar{\lambda}_p = \sqrt{\frac{\sigma_{v,cr}}{\sigma_{v,Ed}}} = \sqrt{\frac{\frac{f_{y,d}}{\sigma_{v,Ed}}}{\frac{\sigma_{v,cr}}{\sigma_{v,Ed}}}} = \sqrt{\frac{f_{y,d}}{\sigma_{v,cr}}} = \sqrt{\frac{f_{y,d}}{\alpha_{cr} \cdot \sigma_{E,cr}}}$$

To be able to calculate the decisive design value of the supporting buckling stress, you must first calculate reduction factors for plate buckling and Eulerian column-type behaviour. For the reduced stresses method, reduction factors are available in [1], chap. 4 and 5, or [1], Annex B.

The reduction functions specified in chap. 4.5.4(1) and 5.3(1) refer each to one of the stress components  $\sigma_x$ ,  $\sigma_z$  and  $\tau_{xz}$ . The reduction factors calculated this way must also be able to take the Eulerian column-type behaviour into account. You can interpolate them on the basis of the comparison stress hypothesis.

$$\left(\frac{\sigma_{x,Ed}}{\rho_x \cdot f_{y,d}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{\rho_z \cdot f_{y,d}}\right)^2 - \left(\frac{\sigma_{x,Ed}}{\rho_x \cdot f_{y,d}}\right) \cdot \left(\frac{\sigma_{z,Ed}}{\rho_z \cdot f_{y,d}}\right) + 3 \cdot \left(\frac{\tau_{xz,Ed}}{\chi_w \cdot f_{y,d}}\right)^2 \leq 1$$

In [15] it is pointed out that Eq. (10.5) partly led to unsafe results in the case of biaxial pressure. The following modification of the interaction equation has been proposed:

$$\left(\frac{\sigma_{x,Ed}}{\rho_x \cdot f_y / \gamma_{M1}}\right)^2 + \left(\frac{\sigma_{z,Ed}}{\rho_z \cdot f_y / \gamma_{M1}}\right)^2 - V \cdot \left(\frac{\sigma_{x,Ed}}{\rho_x \cdot f_y / \gamma_{M1}}\right) \cdot \left(\frac{\sigma_{z,Ed}}{\rho_z \cdot f_y / \gamma_{M1}}\right) + 3 \cdot \left(\frac{\tau_{xz,Ed}}{\chi_x \cdot f_y / \gamma_{M1}}\right)^2 \leq 1$$

With  $V = \rho_x \cdot \rho_z$  if  $\sigma_{x,Ed}$  and  $\sigma_{z,Ed}$  are compressive stresses; otherwise  $V = 1$

The program applies the modified equation in the design according to DIN EN 1993-1-5.

The reduction factor suitable for longitudinal stress is determined in accordance with EN 1993-1-5, chap. 4.5.4. These regulations also apply to transverse stresses that are distributed over the entire longitudinal edge. If loading applies that is not distributed over the entire longitudinal edge, the interpolation of plate-type and Eulerian column-type behaviour will produce incorrect results. In this case, you should refer to Annex B. Shear loading should be handled in accordance with 1993-1-5, chap. 5.3.

## Plate-type behaviour

The following general term applies:

$$\rho_p = \frac{1}{\varphi_p + \sqrt{\varphi_p - \bar{\lambda}_p}}$$

The variables are:

$$\varphi_p = \frac{1}{2} \cdot \left[ 1 + \alpha_p \cdot (\bar{\lambda}_p - \bar{\lambda}_{p0}) + \bar{\lambda}_p \right]$$

$$\bar{\lambda}_p = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}} = \sqrt{\frac{f_{yd}}{\alpha_{cr} \cdot \sigma_{v,Ed}}} \quad \text{referenced system slenderness}$$

The numerical values for  $\alpha_p$  and  $\bar{\lambda}_{p0}$  should be selected as follows:

Cross section	Predominant buckling mode	$\alpha_p$	$\bar{\lambda}_{p0}$
Hot rolled	Longitudinal stresses	$\psi \geq 0$	0.70
		$\psi < 0$	0.13
	Shear stresses Transverse loads		0.80
Welded or cold shaped	Longitudinal stresses	$\psi \geq 0$	0.70
		$\psi < 0$	0.34
	Shear stresses Transverse loads		0.80

Table 1: Numerical values for  $\alpha_p$  and  $\bar{\lambda}_{p0}$ . (Cf. [1], Table B1.1).

If the examined component does not behave like an Eulerian column, the verification can be based on a single degree of system slenderness and the comparison stress hypothesis. In other cases, this method should be applied separately to the different stress components. The resulting reduction factors  $\rho_x$ ,  $\rho_z$  and  $\chi_w$  must subsequently be interpolated with the reduction factors for Eulerian column-type behaviour. (Cf. [1], B.1(3)).

## Eulerian column-type behaviour

The reduction factors for Eulerian column-type behaviour are calculated analogously to the method using effective widths. For longitudinal and transverse stresses, refer to [1], chap. 4.5.3. The interpolation between the reduction factors for plate-type behaviour and the ones for Eulerian column-type behaviour is done in accordance with [1], chap. 4.5.4.

For the elastic Euler buckling stress, the value should be used that results when the supporting conditions of the edges that run in parallel to the stress component are not considered. These are the longitudinal edges with  $\sigma_x$  and the transverse edges with  $\sigma_y$ .

The slenderness ratio for Eulerian column-type behaviour results from the following term:

$$\bar{\lambda}_c = \sqrt{\frac{f_{yd}}{\sigma_{cr,c}}} = \sqrt{\frac{f_{yd}}{\alpha_{cr,c} \cdot \sigma_{Ed}}}$$

The resulting slenderness ratio should be used to calculate the reduction factor for lateral buckling in accordance with EN 1993-1-1, para. 6.3.1.2.

$$\chi_c = \frac{1}{\phi_p + \sqrt{\phi_p^2 - \bar{\lambda}_c^2}} \leq 1$$

The variables are:

$$\phi = \frac{1}{2} \cdot \left[ 1 + \alpha \cdot (\bar{\lambda}_c - 0,2) + \bar{\lambda}_c \right]$$

For unbraced buckling fields, the imperfection coefficient  $\alpha$  should be used in accordance with the Euler buckling stress curve a with  $\alpha = 0.21$ . (Cf. [1], Chap. 4.5.3(5)).

## Interpolation between plate-type and Eulerian column-type behaviour

In order to obtain the reduction factor decisive for the verification, the reduction factors from plate-type behaviour and Eulerian column-type behaviour must be interpolated. The interaction equation is as follows:

$$\rho_c = (\rho_p - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c$$

The variables are:

$$0 \leq \xi = \frac{\sigma_{cr,p}}{\sigma_{cr,c}} - 1 \leq 1$$

The  $\xi$  factor can practically be interpreted as the distance between the buckling stress and the Euler buckling stress.

By limiting the  $\xi$  value, three areas can be defined:

1. The elastic Euler buckling stress is greater than the elastic buckling limit stress.  $\xi$  becomes zero. The failure mode is Eulerian buckling.  
The following is true:  $\rho = \chi_c$
2. The elastic Euler buckling stress is smaller than the elastic buckling limit stress.  $\xi$  ranges in the area  $0 < \xi < 1$ . The failure mode is Eulerian column-type behaviour.  
The following is true:  $\rho_c = (\rho_p - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c$
3. The elastic Euler buckling stress is considerably smaller than the elastic buckling limit stress.  $\xi$  becomes one. The failure mode is plate buckling.  
The following is true:  $\rho = \rho_p$

## Verification process

The verification is based on the comparison stress hypothesis:

$$\left( \frac{\sigma_{x,Ed}}{\rho_x \cdot f_{y,d}} \right)^2 + \left( \frac{\sigma_{z,Ed}}{\rho_z \cdot f_{y,d}} \right)^2 - \left( \frac{\sigma_{x,Ed}}{\rho_x \cdot f_{y,d}} \right) \cdot \left( \frac{\sigma_{z,Ed}}{\rho_z \cdot f_{y,d}} \right) + 3 \cdot \left( \frac{\tau_{xz,Ed}}{\chi_w \cdot f_{y,d}} \right)^2 \leq 1$$

## Requirements on buckling stiffeners

### General

Plates under buckling stress are sheet metal plates with or without stiffeners normally assumed with pinned supports at their edges. The edges of the plate under buckling can be supported elastically or remain unsupported, of course. You should keep in mind, however, that more restrictive regulations of the boundary conditions might be specified in the National Annexes to EN 1993-1-5. The method of reduced stresses as per DIN EN 1993-1-5/NA:2010-12 NDP, 10(1), comment 2, is only permissible if all edges of the plate under buckling stress are assumed as pinned.

In the definition of the minimum requirements applying to buckling stiffeners, it is distinguished between transverse and longitudinal stiffeners. The load-bearing verifications are identical in both cases to the extent that the cross-sectional resistance as well as the buckling safety and torsional buckling safety have to be verified. The effects of actions have to be calculated with respect to different criteria, however.

### Transverse stiffeners

#### Minimum requirements concerning transverse stiffeners

The buckling resistance verification is based on the assumption that the transverse stiffeners for the sheet plate to be verified constitute a rigid support. In order to comply with this assumption, the transverse stiffeners must satisfy particular requirements with respect to their stiffness and strength.

The stiffness requirements can be considered as being satisfied if the area moments of inertia for bending and torsion do not fall below particular limits. A corresponding set of formulas is available in EN 1993-1-5, Para. 9.2.1 (5), (8) and (9). If the verification of the buckling stiffeners is based on a single degree of slenderness of the structural system, the rigidity of the buckling stiffeners is already included in this analysis. When applying the elastic method in the second-order bending torsion analysis and taking initial imperfection into account, evidence must be provided that

- the load-bearing capacity of the equivalent cross section of the buckling stiffeners is ensured and
- the elastic deformation does not exceed the value  $b/300$  in the ultimate limit state

### Cross section and structural system

The stiffener cross section may be assumed with a contributing plate width of  $15\epsilon t$  on both sides, without overlapping, however. If half of the geometric distance to the adjacent transverse stiffeners or buckling plate edge is smaller than  $15\epsilon t$ , the smaller value should be taken into account.

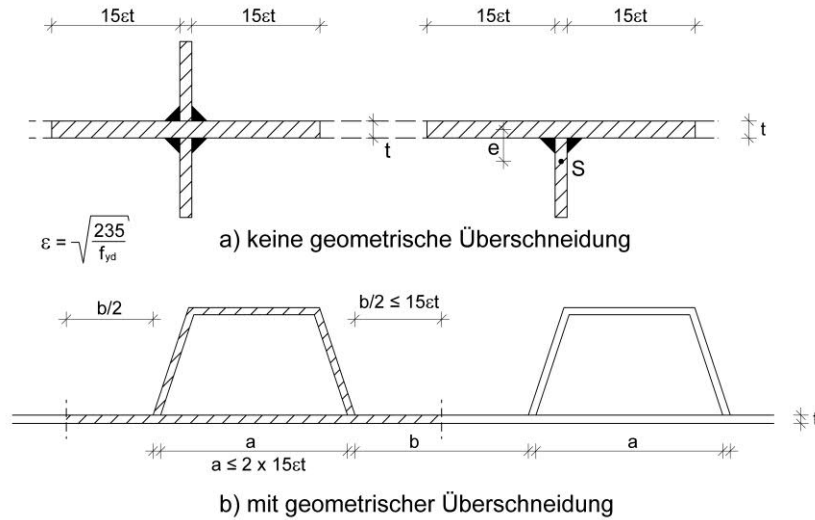


Illustration 1: Effective stiffener cross section for the stiffener verification

You should note in this connection that the stiffener cross section for the verification of the buckling stiffener is different to the one for the verification of braced plates under buckling stress.

The selected structural system is an equivalent member with pinned supports on both sides and with consideration a sine-shaped initial imperfection  $w_0$  with a length  $b$ , which is determined by the distance between the centres of gravity of the respective flanges. It is assumed in this connection that the adjacent transverse stiffeners are not affected by imperfections. It is furthermore assumed that the sheet metal plates and the longitudinal stiffeners, if any, are also connected with a pinned joint to the transverse stiffeners.

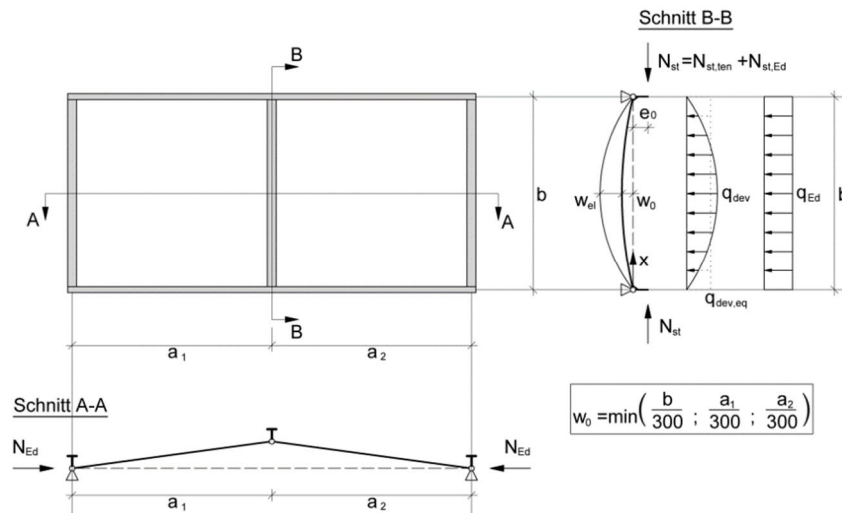


Illustration 2: Structural system for the verification of transverse stiffeners



## Actions

The following actions have to be considered:

1. Deviation forces due to longitudinal axial stresses  $q_{dev}$ .
2. Axial force due to tension field stress or membrane stress caused by shear force action  $N_{st,ten}$ .
3. Axial force produced by external transversal loads  $N_{st,Ed}$ .

### Deviation forces produced by longitudinal axial stresses

Deviation forces caused by longitudinal axial stress are determined on the basis of the imperfection of the transverse stiffeners and the deviation stress in the following expression:

$$q_{dev} = (w_0(z) + w_{el}(z)) \cdot \sigma_m$$

The variables are:

$w_0(z)$       Function of the initial imperfection

$w_{el}(z)$       Function of the elastic deformation resulting from the stiffener calculation

$\sigma_m$           Deviation stresses caused by longitudinal stresses in the adjacent fields

The sine-shaped initial imperfection produces a sine-shaped behaviour of the deviation forces caused by longitudinal stresses  $\sigma_x$ , which can be converted into an equivalent constant line load:

$$q_{dev,eq} = \frac{\pi}{4} \cdot \sigma_m \cdot (w_0 + w_{el}) \quad w_0 \text{ and } w_{el} \text{ are the amplitude values}$$

The equations above reveal that the deviation forces depend on the known development of the initial imperfections as well as on the unknown quantitative development of the elastic deformations. Therefore, an iterative calculation is basically appropriate.

A decisive stiffness criterion for transverse stiffeners is the limitation of the elastic deformation to the value  $b/300$ , however. This value also constitutes the maximum permissible contribution to the amount of deviation forces. For this reason, an iterative approach to the determination of the deviation forces can be dispensed with in accordance with EN 1993-1-5, para. 9.2.1 (7), if  $w_{el}$  is included in the calculation with  $b/300$ . By taking  $w_0$  and  $w_{el}$  with  $b/300$  into account, the calculation can be reduced to a simplified version:

$$q_{dev,eq} = \frac{\pi \cdot b}{600} \cdot \sigma_m$$

### Determination of the deviation stress $\sigma_m$

The deviation stress acting on a transverse stiffener is calculated on the basis of the applying longitudinal compressive force as follows:

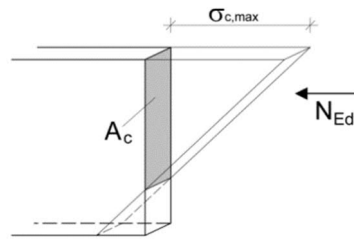
$$\sigma_m = \frac{N_{Ed}}{b} \cdot \left( \frac{1}{a_1} + \frac{1}{a_2} \right) \cdot \frac{\sigma_{cr,c}}{\sigma_{cr,p}} \quad \text{with } 0,5 \leq \frac{\sigma_{cr,c}}{\sigma_{cr,p}} \leq 1$$

The variables are:

$$N_{Ed} = t \cdot \int_0^b \sigma_x(z) \cdot dz \quad \text{provided that the entire transversal edge is under compressive stress, } 0 < \psi$$

$$N_{Ed} = \sigma_{c,max} \cdot \frac{A_c}{2} \quad \text{for stress conditions with } -1 \leq \psi \leq 0$$

$A_c$  the cross-sectional area of the metal plate that is under pressure



$\frac{\sigma_{cr,c}}{\sigma_{cr,p}}$  takes a behaviour similar to the Eulerian column into account

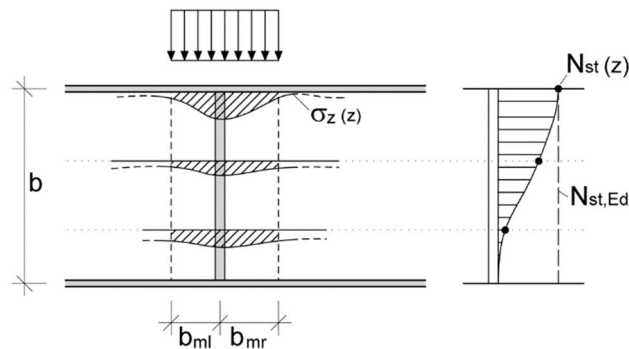
**Axial force from internal loads  $N_{st,ten}$  and external loads  $N_{st,Ed}$**

In general, the applying axial force in the stiffeners is composed of an external portion  $N_{st,Ed}$  and an additional internal portion  $N_{st,ten}$ .  $N_{st,Ed}$  results from the direct action of external loads such as the supporting forces transferred to the support stiffeners. In PLII+, these forces are mapped rather realistically by the elastic FE panel calculation with discrete stiffeners.

Due to the rigid coupling of the nodes, the elongations in the stiffeners and the plates along the contact line are identical. Therefore, the axial forces in the stiffeners can be calculated via the extensional stiffness of the beam elements. Consequently, in each section along the stiffener, the axial force in the calculated equivalent cross section is composed of the product of the axial force and the net area of the stiffener and of the integral of the axial stresses over the contributing plate areas on both sides of the stiffener (see also illustration 3):

$$N_{St,Ed} = A_{St,netto} \cdot \sigma_{z,x=0} + \int_{b_m} \sigma_{z,x} \cdot t dx$$

Thanks to this generally accepted approach, a distinction between support stiffeners and intermediate stiffeners is not required, because the plate's stress field on which this approach is based includes the effects of all loads. To facilitate comprehension and tracing, the maximum axial compressive force is taken into account in the verification of the stiffeners. The result of this calculation deviates only slightly from when taking the actual axial force behaviour into account.



*Illustration 3: Determination of the axial stiffener force  $N_{st,Ed}$  resulting from the axial plate stresses  $\sigma_z$*

The internal portion of the axial force is determined by the mechanism that is generated by the transfer of the overcritical shear stresses. The shear forces are transferred via the shear stresses in the plate down to the smallest buckling load. The portion that exceeds this stress level brings about a tension field, i.e. axial tension stresses that act diagonally over the shear panel (mental concept of drapery). The circumferential stiffeners function in this model as compression-loaded upright members.

The internal axial compressive force can be calculated in a strongly simplified way using the equation:

$$N_{st,ten} = V_{Ed} - \frac{1}{\lambda_w^2} \cdot t \cdot h_w \cdot \frac{f_y}{\gamma_{M1} \sqrt{3}} > 0$$

$V_{Ed}$  stands for the highest shear force in the adjacent plates under buckling stress. As this estimation is far on the safe side, the verified cross section may be assumed at a distance of max. 50 % of the shear panel height from the point where the max. shear force loading applies. Supported by FE calculations, the shear force can be determined with the help of the integral of the shear stresses along the verified cross section.

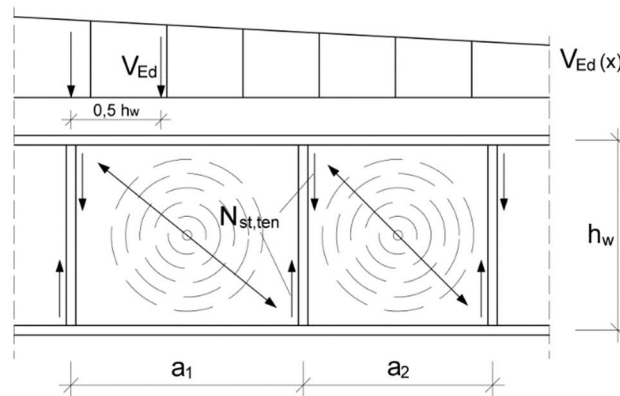


Illustration 4: Assumptions for the calculation of  $N_{st,ten}$

If the stiffeners are arranged on one side or asymmetrically, they produce an offset between the central plate plane being the assumed action plane of the axial plate forces and the centre of gravity of the calculated stiffener cross section. This eccentricity  $e_0$  is to be taken into account.

### Longitudinal stiffeners

The cross-sectional bearing capacity of the longitudinal stiffeners is also to be verified. The verification is similar to that of the transverse stiffeners. If longitudinal stiffeners are placed between transverse stiffeners, the latter may be assumed as rigid pinned supports. The length of the equivalent compression member corresponds to the spacing of the respective transverse stiffeners.

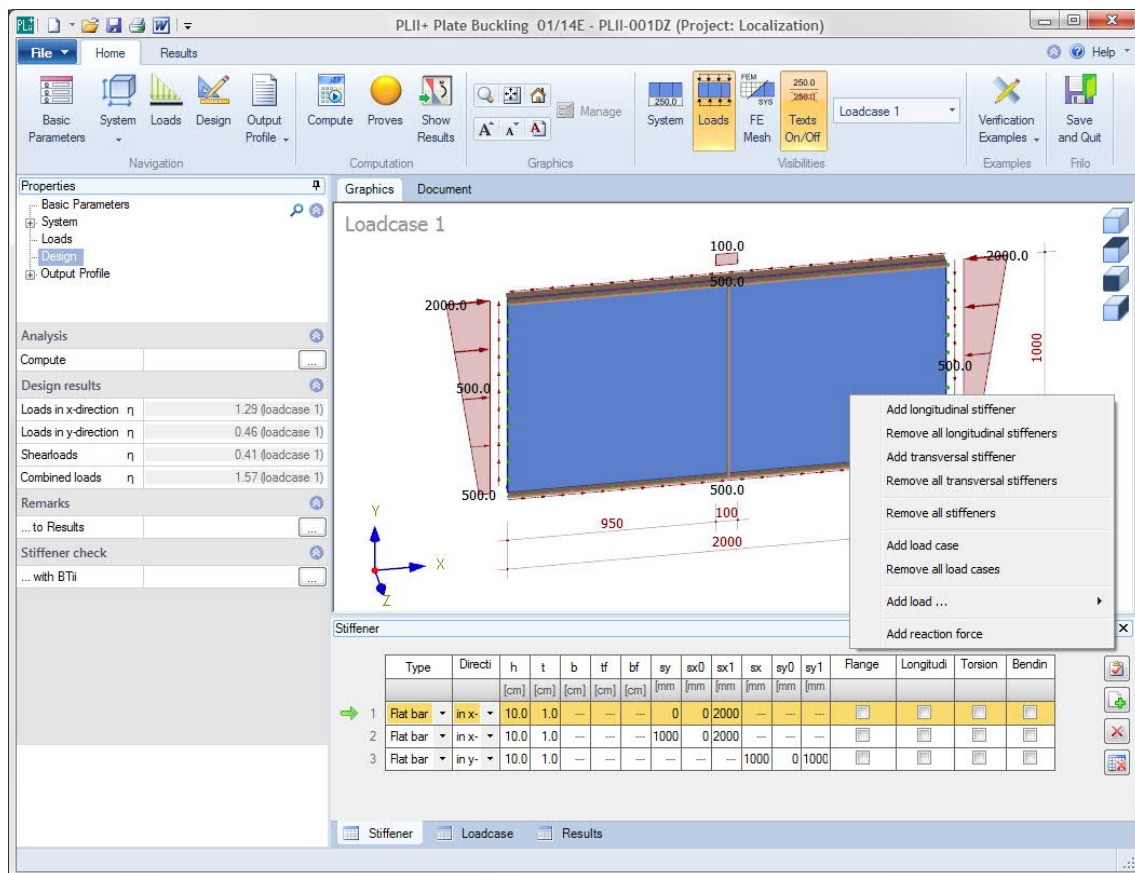
## Definition of the structural system

Help texts and information about each individual definition value are an integral part of the user interface. Clicking in an input field displays a description of the corresponding value in the information area. Therefore, this manual only describes definition values that are not thoroughly explained in the user interface.

## User interface

A general description of the user interface is available in the document:

[Basic operating instructions](#)



### Interactive 3D GUI PLII+

The central feature is the three-dimensional graphical user interface. All entered values are immediately shown on the graphic screen true to scale. When you keep the mouse cursor on a graphical element for a short while, a tooltip with information about properties is displayed. Clicking on a value enables the associated input field. Right clicking displays the context menu which offers a range of useful functions (e.g. adding of buckling stiffeners or loads/load cases when right clicking on the plate, or removal of a load/buckling stiffener when right-clicking on the load or stiffener).

### Definition via tables

All list-type definitions (e.g. buckling stiffeners) can also be entered via tables. Clicking on a tap at the bottom of the screen displays the corresponding table. Buttons for the row management (adding, deleting, etc.) are available at the right table border.

See also the Basic Operating Instructions FDC-MFL.pdf ▶ [Definition via tables](#)

### Selection of the graphical representation

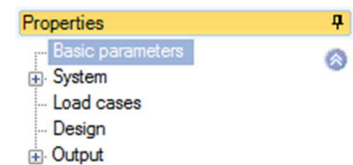
You can toggle between different graphical representations, e. g. load case, system and loading.

*Note: After starting PLII+ or opening an item, the system is displayed as a default with the dimensions of the buckling field and the buckling stiffeners. Only after switching to the representation of the loading, all defined loads for the selected load case are displayed together with the dimensions of the concentrated loads.*

## Basic parameters

### Calculation mode

As a standard, the software always performs the bearing strength verification in accordance with the selected code. If the verification is not possible due to limitations by the code, e. g. number of supported edges or a similar restriction, you can switch to the analysis mode in order to be able to calculate at least the buckling factors and buckling coefficients in accordance with the linear buckling theory.











### Bending caused by concentrated loads

You can find a description in the chapter [Application options](#), and particularly, in the paragraph [Load cases and edge loads](#).

### Nominal stresses of the edge loads

In general, the Finite Elements Method is an approximation method the accuracy of which increases with the mesh fineness. This is particularly true for the membrane stresses under concentrated loads, whereas the calculated buckling loads depend less on the fineness of the mesh. Therefore, the software optionally allows you to perform the verification (verification equation determined by the selected code) with the stresses obtained from the nominal values of the edge loads. With a concentrated load, the nominal stress obtained from the amount and the contact area is used in the calculation instead of the plate stress obtained in the FE calculation which is normally lower.

Standard and safety concept		
Design standard	BS EN 1993-1-5:2008	
Analysis modes		
Analysis mode	Design check	
Bending due to patch loads	exclude	
Design		
Nominal values of loads	<input checked="" type="checkbox"/>	
Mesh parameter		
Mesh options	Number	
Number of elements		288

## System - buckling stiffeners

### Panel boundary conditions

A pre-setting of the panel boundary condition is not provided for, because the edge loads are always in equilibrium and/or equilibrium forces are internally applied to the vertical edges.

### Supporting conditions

The plate boundary conditions refer to displacement  $w$  in the  $z$ -direction, torsion  $\varphi_x$  around the  $x$ -axis and  $\varphi_y$  around the  $y$ -axis as well as warping  $\varphi_{xy}$  at the nodes.

PLII+ allows the following types of edge supports, verifiability must be observed:

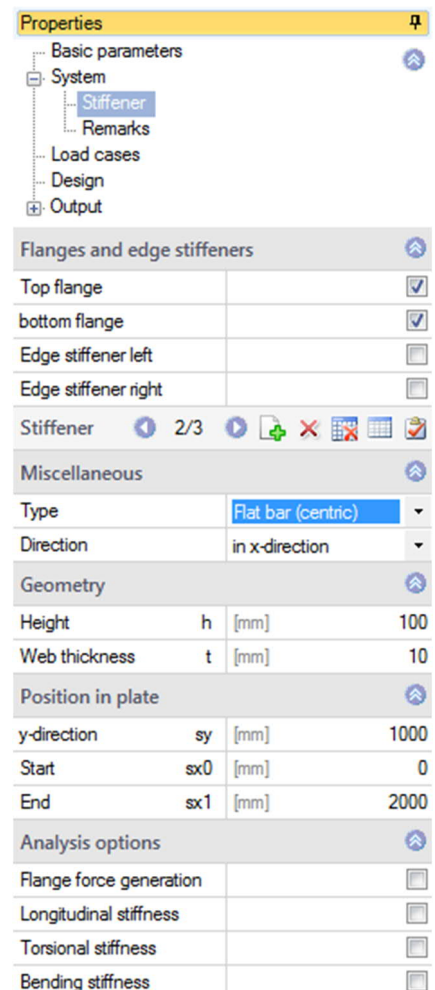
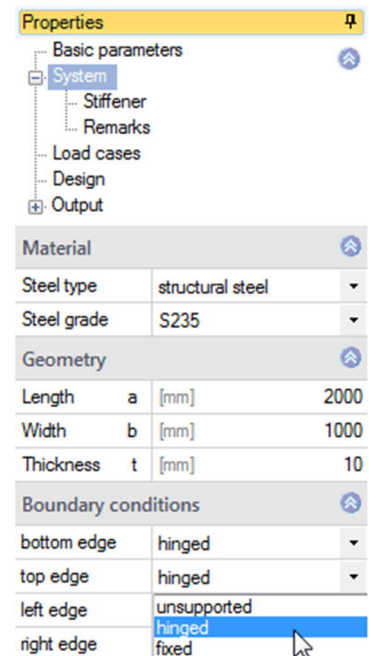
- |                  |  |
|------------------|--|
| Free edge:       | Displacement $w$ perpendicular to the plate; bending torsion $\varphi_x$ and $\varphi_y$ and warping $\varphi_{xy}$ of one edge are not impeded.                           |
| Pinned edge      | (pinned support free from moments):<br>Displacement $w$ and torsion $\varphi$ perpendicular to the edge are suppressed, torsion around the edge axis and warping are free. |
| Restrained edge: | Displacement $w$ , torsion $\varphi_x$ , $\varphi_y$ and warping $\varphi_{xy}$ are suppressed on all nodes of an edge.  |

### Flanges and edge stiffeners

The available four options allow the user to define comfortably stiffeners on the four plate edges, e.g. bottom and top flange of an I-section beam. When setting a checkmark, a flat steel stiffener is generated over the entire length of the plate edge in question. The cross section of the stiffener must be adjusted appropriately - see [stiffness geometry - type](#). The individual stiffeners can be selected and displayed via the [table toolbar](#). If you remove the checkmark, the associated buckling stiffener is removed from the list.

### Calculation parameters

- |                  |  |
|------------------|--|
| Flange forces:   | If you check this option, additional flange forces are generated by the software to prevent the discharge of the edge loads into the stiffeners. You can only generate flange forces on stiffeners that run over the total length or width of the plate.   |
| Axial stiffness: | When you check this option, the software takes axial stiffness into account in the panel calculation.<br><br>If the axial stiffness of the stiffeners is not considered in the examination of the panel (typical in steel engineering), PLII+ calculates the stiffener forces for the buckling examination from the strain in the panel at the location of the stiffeners. The stiffener forces calculated this way are used in the geometric stiffness matrix as drive forces if compression applies and as stabilizing forces if tension applies. The resulting buckling values are identical to those in [3] and [4] (see also <a href="#">ideal buckling values</a> → p. 9). |



When taking the elastic stiffness of the stiffeners into account in the panel examination, the edge loading is redistributed particularly in the area of the stiffeners. The resulting stiffener forces may considerably differ from those calculated without consideration of the stiffener stiffness. Under normal conditions, the buckling values calculated from the different stiffnesses vary only slightly, however.

The consideration of the axial stiffness of flange stiffeners causes the increase of the bending stiffness around the web normal, in particular, if wheel or bearing loads apply at the longitudinal buckling field edges. As a consequence, the local loading is reduced. The ideal buckling values for this load case increase considerably. If edge loads apply simultaneously to the transverse edges due to  $\sigma_x$ -stress from bending moment and axial force, you must make sure that a stress redistribution through discharge of the longitudinal stresses into the flanges does not take place. You can prevent this effect by activating the "Flange forces" option. In this case, PLII+ generates concentrated flange forces at the ends for the edge loading  $r_x$  by multiplying the edge stress at the level of the flanges with the associated cross section area of the stiffeners (flanges). A redistribution of the  $r_x$  loading is excluded this way. You should note in this connection that the first eigenvalue normally drops when the axial stiffness of transverse stiffeners on I-section beams loaded on their longitudinal edges is neglected. This is due to the fact that the transverse stiffener is loaded by compression forces that result from the panel calculation without stiffener and, therefore, further affect the stability of the entire system. Consequently, you should take the axial stiffness of the transverse stiffeners into account in the panel calculation to solve these problems correctly.

**Torsional stiffness:** When you check this option, the software takes torsional stiffness into account in the buckling examination.

In order to reduce the number of parameters while staying on the safe side, the torsional stiffness of the stiffeners is neglected in the buckling values given by reference [3] and [4].

As you can see from the examples ([ideal buckling values](#) → p. 9), the consideration of torsional stiffness produces a considerably higher buckling stress, a lower buckling slenderness and an increased buckling strength, particularly in combination with closed trapezoidal stiffeners. The consideration of the torsional stiffness is enabled by default. You can disable this option when defining the stiffeners. The torsional buckling (torsional rotation around the longitudinal axis) under centric compressive stress is taken into account.

**Bending stiffness:** Stiffness around the plate normal. When you check this option, the software takes the stiffness around the plate normal into account in the buckling examination.

Basically, you can further increase the buckling strength by taking the bending stiffness of the stiffeners around the plate normal into account. The bending stiffness results from the load-distributing effect of longitudinal stiffeners, in particular with load introduction problems.

*Note:* As a standard, all stiffness portions are taken into account, because they are effective in reality due to the weld connection. The deactivation is only necessary in special cases, e.g. if manual calculations have to be considered subsequently.

Stiffener geometry - type

Flat steel, centric	Flat steel, eccentric
Angular stiffener	T-stiffener
Trapezoidal stiffener	



## Load cases

Operation of the load cases menu: see also [table toolbar](#)

### Equilibrium forces

If the defined edge loads are not in equilibrium (e.g. with concentrated loads), equilibrium forces are automatically generated by the software. You can optionally select whether shear forces should be generated at both vertical edges (corresponds to a pinned beam support) or shear forces and a fixed-end moment should be generated at a single vertical plate edge (corresponds to a cantilever). As a consequence, variable longitudinal stresses occur over the plate length, which can be used for the buckling analysis and the verification process.

### Support reaction

PLII+ allows you to define a concentrated load at the lower plate edge that counteracts the concentrated loads at the upper edge in the sense of a support reaction. If the concentrated loads and the support reaction are not in equilibrium, the software generates internally equilibrium forces at the vertical plate edges. This allows you to map simple two-span beams. The main application case of this software is the examination of the situation where a wheel (pair) is above a support, however.

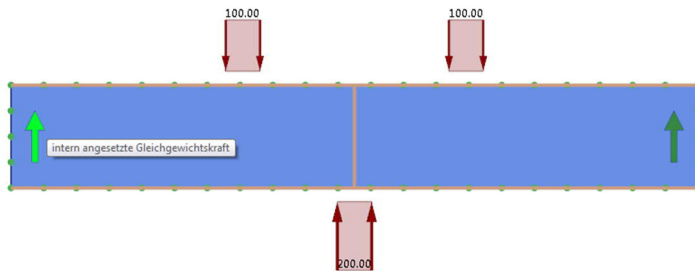


Illustration: Loading by a pair of wheels above a support with internally generated equilibrium forces

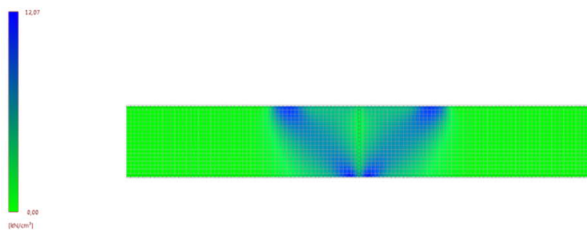


Illustration: Associated comparison stresses

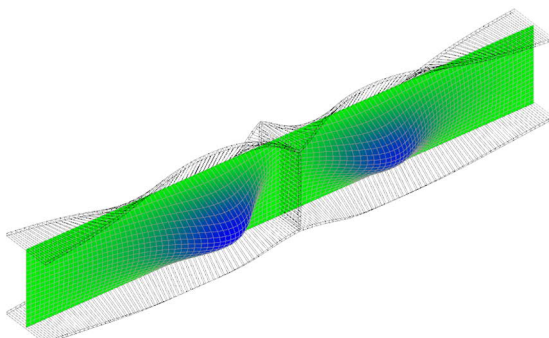


Illustration: Associated buckling pattern

**Properties**

- Basic Parameters
- System
- Loads
- Design
- Output Profile

---

**Loads**

Loadcase 1/2

Miscellaneous

Name

Remarks to the loads

Loads in x-direction

Load value at y=0	$r_x(y=0)$	[N/mm]	500.0
Load value at y=b	$r_x(y=b)$	[N/mm]	2000.0

Loads in y-direction

Load type	Patch loads (+ equilibrium)		
balance forces	linear distributed loads		
first patch load	$R_y,1,d$	[kN]	100.0
Distance	x1	[mm]	1000
Length of loaded patch	c1	[mm]	100
Second patch load	$R_y,2,d$	[kN]	0.0
Distance	x2	[mm]	0
Length of loaded patch	c2	[mm]	0
Support present	<input type="checkbox"/>		

Shearloads

Load value	$r_{x,y}$	[N/mm]	500.0
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## Reference literature

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