BTII+: Additional notes concerning practical application

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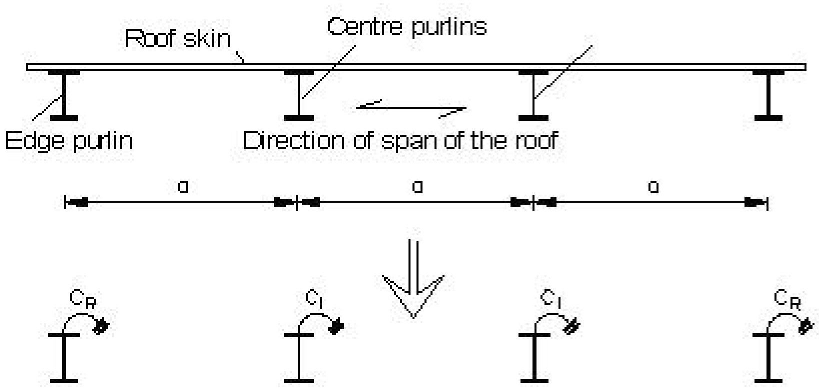
Purlins with torsionally elastic support by the roof skin



ED: modulus of elasticity of the roof skin

ID: moment of inertia of the roof skin per length unit

The transfer of the moment between the purlin and the roof due to contact or loading on the connectors is to be verified. See also Vogel/Heil [13].

If the moment to be transferred exceeds the contact moment (= created by the drift of the load application point to the flange edge), the compliance of the connection between the purlin and the trapezoidal sheet must be taken into account in addition. See also Lindner [5].

Trusses with torsionally elastic support by purlins

The spring stiffnesses are calculated as described above. It must be distinguished between centre trusses and edge trusses.

Trusses with elastic translational support at the top chord by purlins

The stiffness of the horizontal equivalent spring results from the compliance of the horizontal roof structure in the edge spans. If required, also the slip in the connectors must be taken into account.

For more information concerning the calculation of equivalent stiffnesses in different types of structural framework, see Rubin/Vogel [12], for instance.

Trusses with elastic torsional support by columns

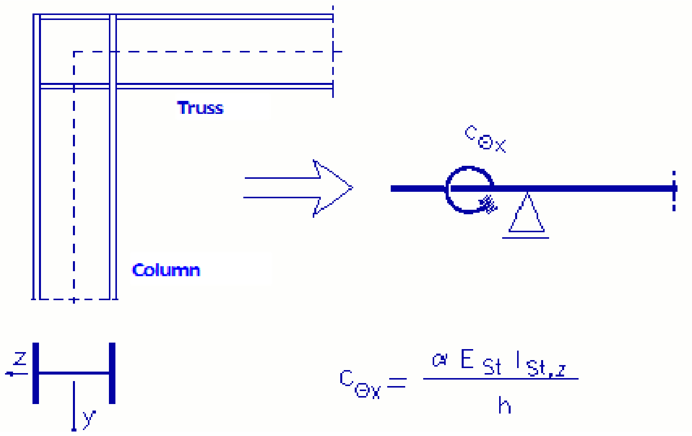
E_{St} modulus of elasticity of the column

$I_{St,z}$ moment of inertia of the column around the z-axis

h height of the column section

α restraint value depending on the support of the column base around the weak axis.
α = 4: restrained
α = 3: pinned

The supporting effect is low under normal conditions.



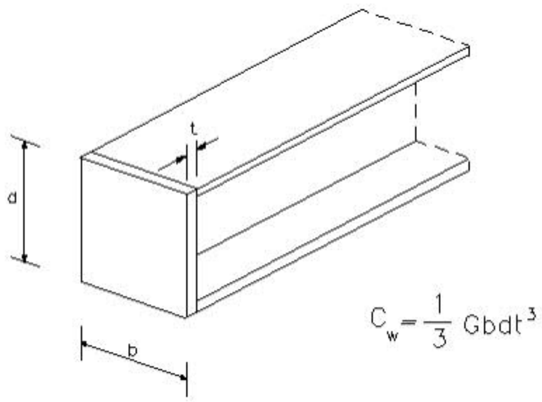
$$c_{\Theta x} = \frac{\alpha \, E_{St} \, I_{St,z}}{h}$$

Beam with elastic warping support

The free warping fixity increases the torsional stiffness of beams with thin-walled open cross sections. In the following, we are going to give you some information about the calculation of discrete warping springs C_w in three frequent cases of warping fixity.

a) End plate

G shear modulus of the end plate's material



$$C_w = \frac{1}{3} G b d t^3$$

b) Projection (can directly be modelled!)

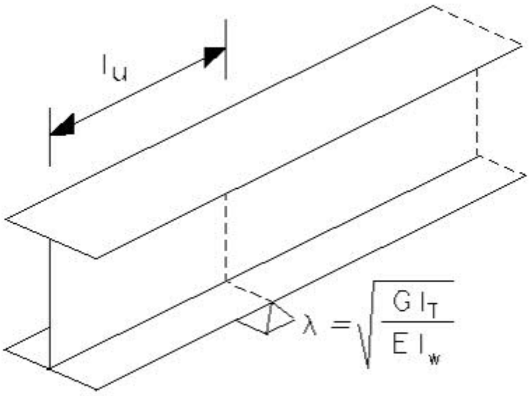
I_T Saint-Venant's torsional moment of inertia

I_ω warping moment of inertia

l_u projection

E modulus of elasticity

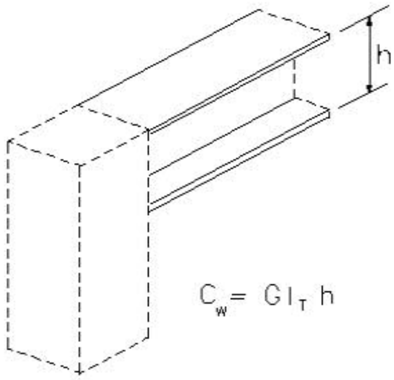
G shear modulus



$$\lambda = \sqrt{\frac{G I_T}{E I_w}}$$

$$C_w = G I_T \frac{1}{\lambda} \tanh(\lambda l_u)$$

c) Column connection



h	beam height (distance between the centres of gravity of the flanges)

I_T	Saint-Venant's torsional moment of inertia of the column

Open sections

$$I_T = \sum\left(\frac{1}{3}\cdot s_i \cdot t_i^3\right)$$

Closed sections

$$I_T = 4\cdot\frac{A_m^2}{\sum\frac{s_i}{t_i}}$$

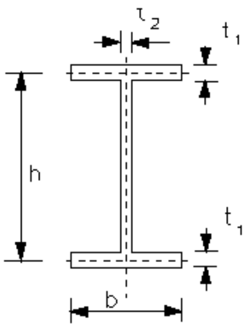
The variables refer to:

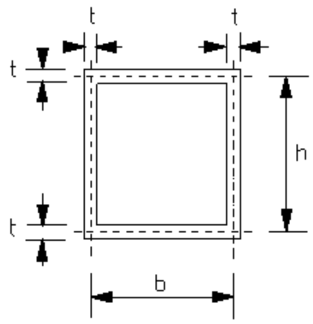
s_i	length of the i[th] rectangle

t_i	width of the i[th] rectangle

A_m	surface of the cross section enclosed by the section centre line

Examples





$$I_T = \frac{1}{3}(2bt_1^3 + ht_2^3) \qquad I_T = 4b^2h^2t/(2(b+h))$$

Beam with shear field support

Purlins under roof plates are supported rigidly or elastically against lateral shift through the shear field stiffness at the height of the top chord. In the current version, *BTII* does not provide any options to describe the shear field effect exactly.

An approach by approximation can be achieved by converting the shear field stiffness S* to an equivalent elastic foundation with the stiffness Cy applying at the top chord $\bar{c}_y$

The conversion is obtained by setting the virtual work of the elastic foundation equal to that of the shear field.

$$\int S^* v_o' \delta v_o' dx = \int \bar{c}_y v_o \delta v_o dx$$

When assuming a sinusoidal horizontal shift of the top chord with n half-waves over the length of the beam

$$v_o = \bar{v}_o \sin \frac{n \pi x}{l}$$

it follows

$$\bar{c}_y = S^* \left(\frac{n \pi}{l}\right)^2$$

First, perform the calculation of the elastic foundation $\bar{c}_y$ with n = 1. Verify subsequently the elastic foundation on the basis of the shift of the top chord caused through this and/or repeat the calculation with n>1.

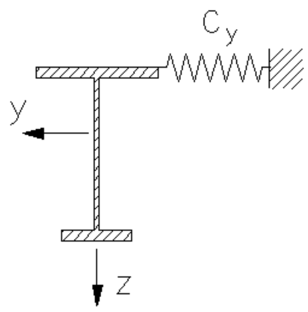
This approximation is sufficient in many cases of practical application.

Lateral torsional buckling with a fixed axis of rotation

The problem of lateral torsional buckling with a fixed axis of rotation at a distance z from the shear centre often occurs in practice. You can describe it in *BTII* as follows:

Define an elastic translational foundation in y-direction with a *stiffness 10E+8 to 10E+10* at the distance z_0 from the centre of gravity. The resulting shift and torsion in regard to the centre of gravity and the shear centre are equal to zero along the pre-set fixed axis of rotation.

You can also use eccentric discrete springs to provide fixity against lateral shift in the y- or z-direction at any point in the cross section. For this purpose, you ought to define *high but not too high spring stiffnesses*. As a rule, *the stiffnesses should be < 10E+16* . In order to ensure the numerical stability of the calculation, discrete stiffnesses intended as shift fixities should not be greater than strictly necessary. You can check this by verifying the kinematic constraint conditions in the cross section.



Torsion with solid cross sections

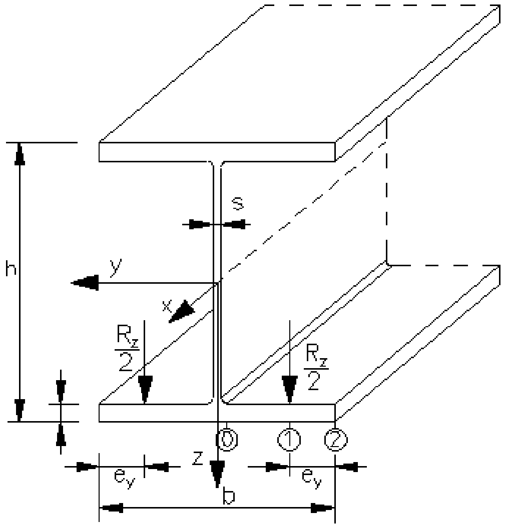
The calculation of beams with solid cross sections such as glued laminated girders or pre-stressed concrete girders requires particular attention in regard to the load distribution of torsional loading.

On thin-walled open cross sections, typical in steel construction, the load is distributed via Saint-Venant's torsion and warping torsion (cross sectional warping fixity), whereby the distribution depends mainly on the beam length and the type of loading.

On solid cross sections, typical in reinforced concrete and timber construction, the load portion distributed via warping torsion is very low and can therefore be neglected. In *BTII* you can take this fact into account by setting the warping moment of inertia of the cross sections to zero, when entering the data. In this case, the warping bimoments determined in the calculation of the internal forces acting on the bar end are equal to zero.

Stresses due to local beam loading

When flange bending stresses apply due to the operation of underslung overhead cranes, the global stresses calculated in accordance with the beam bending theory must be superimposed with the local bending stresses. *BTII* handles this calculation as described in the notes concerning the calculation in reference [1].



The superposition of global and local stresses is limited to the node points where the point loads apply to the flanges. It is analysed separately for each flange side, the top and bottom edge of the flange at the web/flange transition (points 3/4), at the load application points (points 2/5) as well as at the flange edges (points 1/6).

The variable flange thickness of cross sections with inclined flanges (section type 5) can optionally be taken into account. According to [1], local stresses in the length direction of the beam σ_x can be reduced by 75 % before superimposing them with the axial beam stresses. The comparison stresses in the specified points in accordance with the von-Mises yield criterion can optionally be calculated with or without consideration of shear stresses resulting from the Saint-Venant's torsional moment portion.

Lateral buckling of frame systems

Problem

The equivalent bar method as per DIN 18800-2 is an alternative calculation method for the verification of load-bearing systems in second order analyses with inclusion of deformations. This simplified verification is based on ideal bifurcation loads, which are calculated on the straight beam in *BTII*. The calculation of the ideal bifurcation loads is performed separately for each of the failure modes lateral buckling and lateral torsional buckling.

This approach has proven its worth for beam and column systems that typically comply with the Euler bucking modes. Where frame systems are concerned, the calculation is often based on second order analyses. The second order analysis provides for lateral buckling in the plane of the frame under normal conditions. The lateral torsional buckling failure mode must be examined separately, however. This examination is based on a simplified verification in accordance with the equivalent bar method.

Equivalent bars for lateral torsional buckling analyses

In order to verify a bar in a sway or non-sway frame system in accordance with the equivalent bar method, you have to extract it from the total system A single-span beam with fork supports is assumed for the examination of the lateral torsional buckling failure mode. The bar end moments, which result from the calculation of the basic frame in a first or second order analysis, are applied to the single-span beam in accordance with the behaviour of the internal forces. The span moments can be calculated in first-order analyses. The load bifurcation factor is calculated numerically for the structural system generated this way producing the basic value M_{kiy} for the equivalent bar method.

Equivalent bars for lateral buckling examinations

Under normal conditions, the verification of the lateral stability of frame systems is included in the second order calculation of the internal forces. The simplified verification in accordance with the equivalent bar method is much more difficult in this case because you have to adjust the structural system of the equivalent bar via load conditions in such a manner that the effective length corresponds to that of the entire system. To do this, you have to calculate corresponding spring stiffnesses. The equivalent bar verification for lateral buckling requires a system modified in the described way.

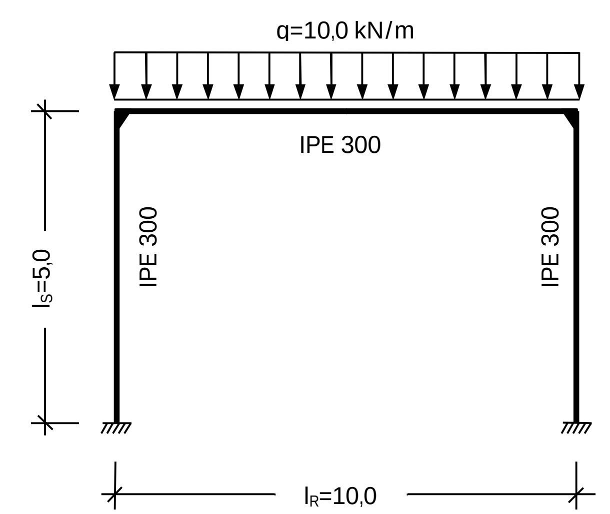
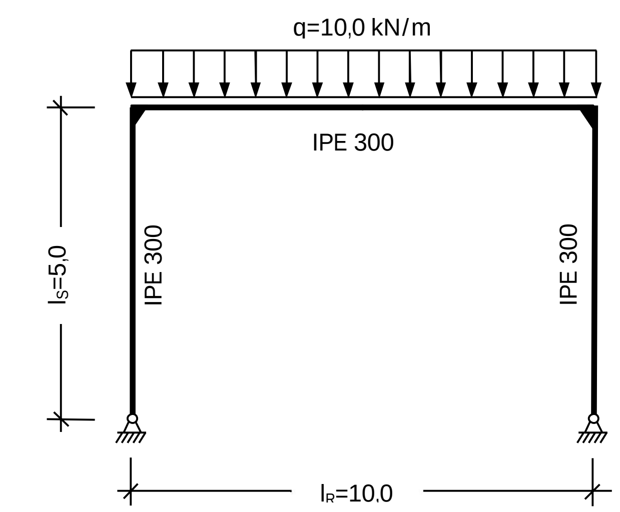
It is particularly difficult to determine the corresponding rotational and translations springs and requires the consultation of expert literature. This verification method and the preparative work involved are quite elaborate in comparison to the calculation of the frame system in second order analyses.

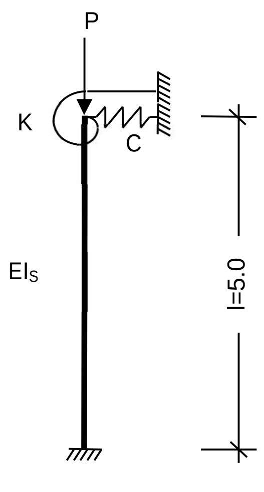
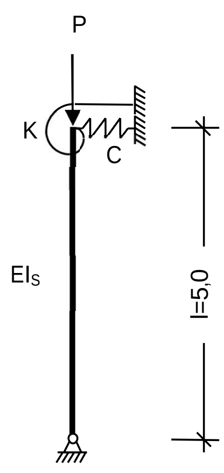
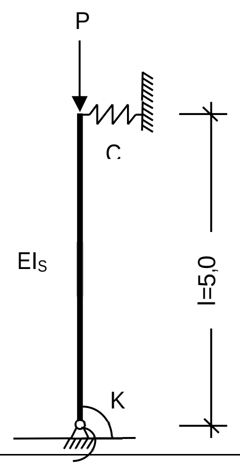
In the following examples, the lateral buckling stability of the frame columns will be examined. You will learn how to calculate the torsion spring constant and which structural system you have to enter in *BTII*.

Example: pinned and restrained frame

Taken from Petersen: Statik und Stabilität der Baukonstruktionen, 2nd edition, 1982, Publ. Vieweg-Verlag, p. 340, table 5.3.

In the present example, the lateral buckling stability of the frame column is examined.

	
$C = 0(\delta = 0)$ single-storey sway frames with restrained column bases.	$C = 0(\delta = 0)$ single-storey sway frames with pinned column bases.

Parameters

$$\delta = \frac{Cl^3}{EI_S}\;;\;\gamma = \frac{K \cdot l_S}{EI_S}$$

Effective length

$$s_K = \beta \cdot l_S$$

Formulas

$$\frac{1}{\gamma} = \frac{1}{6} \cdot \frac{EI_S}{EI_R} \cdot \frac{l_R}{l_S} \qquad \frac{EI_S}{K \cdot l_S} = \frac{1}{6} \cdot \frac{EI_S}{EI_R} \cdot \frac{l_R}{l_S} \qquad \frac{K \cdot l_S}{EI_S} = \frac{6 \cdot EI_R}{EI_S} \cdot \frac{l_S}{l_R} \qquad Kl_S = \frac{6 \cdot EI_R}{l_R}$$

Torsion spring constant

$$K = \frac{6 \cdot 21000 \cdot 8360}{1000} = \frac{3 \cdot 21000 \cdot 8360}{500} = 10533 KNm / rad$$

Reference literature

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[2] W.F. Chen, T. Atsuta: *Theory of Beam- Columns. Vol. 2: Space Behaviour and Design,* McGraw-Hill, New York 1976.

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