Analyses on reinforced concrete cross sections

This document includes additional information about our reinforced concrete software applications.

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Standards and acronyms

EN:	Recommended values EN 1992-1-1 EN 1992-1-1:2004 /A1:2014 and EN 1992-1-2:2004 /AC:2008
NDP	Parameter defined in the National Annex (NA).

Current versions of the National Annexes (NA):

NA-D:	Germany DIN 1992-1-1/ NA:2015-09 and DIN EN 1992-1-2/NA:2015-09
NA-A:	Austria ÖNORM B 1992-1-1:2011 and ÖNORM B 1992-1-2:2011 These NAs replace those of 2007 applicable recently.
NA-GB:	Great Britain NA to BS EN 1992-1-1 A2:2015-07, BS8500-1:2015 and NA to BS EN 1992-1-2:2004
NA-I	Italy UNI EN 1992-1-1/NTC:2008 and EN 1992-1-2:2004 /AC:2008 NTC: The application of Eurocode in Italy is described in the "Norme tecniche per le costruzioni" (/ 56 /) and the supplementary circular "Circolare finissima 2.2.2009" (/ 57 /).
NA-PL	Poland PN EN 1992-1-1:2008/NA:2010 and PN-EN 1992-1-2:2008/NA:2010

Design for bending and longitudinal force

In the design of reinforced concrete, the strain state causing failure is calculated for the given internal forces while the reinforcement is unknown.

Due to the strain distributions in the ULS defined in the standards, at least one border strain is always known. The internal and external forces must be in balance.

The result is two or, with double bending, three non-linear equations, whereby the internal forces are functions of the border strains and the inclination angle of the neutral axis (double bending). They are resolved by iteration with the help of the Newton method.

You can select among the kh-(kd)-method (only with uniaxial loading) or the method with a given reinforcement ratio for the bending design.

Where cross sections under low loading are concerned, compliance with the minimum reinforcement (compression/bending) can become decisive.

In addition, the application indicates when the permissible maximum reinforcement is exceeded.

Bases of design

Internal action curve of concrete	Figure 3.3
Maximum strain f_{cd}	$\alpha_{cc} \cdot f_{ck} / \gamma_c$
Compressive limit strain of concrete ε_{cu2}	ε_{cu2} = 3.5 ‰, > C50 irrespective of type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Compressive strain at end of parabolic area ε_{c2}	ε_{c2} = 2.0 ‰, > C50 irrespective of type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Exponent n	n =2 > C50 depending on type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Internal action curve for reinforcing steel	Figure 3.8
Maximum strain f_{td}	$K \cdot f_{yk} / \gamma_s$
Limit strain of steelε_{ud}	NDP
Strain distribution ULS	Figure 6.1

The stress-strain curve of the concrete corresponds to the parabola rectangle stress diagram.

For standard concrete εc2 = 2 $^{o/oo}$ and exponent = 2, closed formulas (/2/) can be used to calculate the internal forces on rectangular or circular cross sections.

In all other cases (high-performance concrete, T-beams and layers cross sections), an approximation calculation is required by splitting the concrete compression zone in thin layers. With cast-in-place complements, the internal forces of the concrete are calculated using the corresponding internal action curves of the different types of concrete used.

You can optionally take the area of the concrete displaced by the steel in the compression zone into consideration (→ B2 design configuration). The disregard of certain parameters in connection with highly reinforced cross-sections particularly of high-strength concrete, which was common until, recently is no longer justified according to /10/ p. 13.

f_{ck} Characteristic compressive cylinder strength

Strength classes acc. to table 3.1

αcc coefficient for long-term effect NDP

NDP	Standard concrete 3.1.6	Lightweight concrete 11.3.5	Unreinforced 12.3.1
EN	1.0	0.85	0.85
NA-D	0.85	0.75	0.70
NA-GB	0.85	= EN	= EN
NA-A	= EN	= EN	= EN
NA-I	0.85	= EN	= EN
NA-PL	= EN	= EN	= EN

γc partial safety coefficients for concrete NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.5	1.2	1.5
NA-D	= EN	1.3	1.5
NA-GB	= EN	= EN	= EN
NA-A	= EN	= EN	= 1.3
NA-I	= EN	1.0	= EN
NA-PL	1.4	= EN	1.4

Possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control γc,Red1	A2.2 (1) measured or reduced geometric data γc,Red2	A2,2 (2) variation coefficient of concrete strength < 10 % γc,Red3	A2.3 concrete strength in the mixing plant determines the diminishing factor η (γc,Red* η)	A2.3 Minimum γc γc,Red4)
EN	1.4	1.45	1.35	0.85	1.30
NA-D	1.5	1.5	1.5	0.9	1.35
NA-GB	= EN	= EN	= EN	= EN	= EN
NA-A	= EN	= EN	= EN	= EN	= EN
NA-I	1.4	Not allowed	Not allowed	Not allowed	1.4
NA-PL	1.35	Not allowed	Not allowed	Not allowed	1.35

Stress strain curve reinforcing steel:

E_s: E-Module	200000 N/mm^2 or according to approval
f_{yd}: Design value of the yield strength	f_{yk}/γ_s
ε_{yd}: Strain at the design value of the yield strength	f_{yd}/E_s
ε_{uk}: characteristic value of the limit strain	according to ductility
ε_{ud}: Design value of the limit strain	NDP
f_{td} : Design value of tensile strength at ε_{uk}	$K \cdot f_{yk}/\gamma_s$ K according to ductility
$f_{td,cal}$: : Design value of tensile strength at ε_{ud}	determined accordingly ε_{ud}

f_{yk} Characteristic value of the yield strength

f_{tk} $k \cdot f_{yk}$ characteristic value of the tensile strength

Ductility A: k= 1,05 ε_{uk}= 25 o/oo

Ductility B: k= 1,08 ε_{uk}= 50 o/oo

Ductility C: k= 1,15 ε_{uk}= 75 o/oo

ε_{ud}: limit strain NDP

	Permanent / temporary. 2.4.2.4
EN	0,9* ε_{uk}
NA-D	25 o/oo
NA-GB	= EN
NA-A	= EN
NA-I	=EN
NA-PL	=EN

γ_s: partial safety coefficients for reinforcing steel NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.15	1.0	1.15
NA-D	= EN	= EN	= EN
NA-GB	= EN	= EN	= EN
NA-A	= EN	= EN	= 1.0
NA-I	= EN	= EN	= EN
NA-PL	= EN	= EN	= EN

Possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control $\gamma s,Red1$	A2.2 (1) measured or diminished geometric data $\gamma c,Red2$
NA-EN	1.10	1.05
NA-D	1.15	1.15
NA-GB	= EN	= EN
NA-A	= EN	= EN
NA-I	Impossible	Impossible
NA-PL	= EN	= EN

The inclination of the upper branch of the internal action curve of the reinforcing steel is taken into account, unless you have unticked this option in the B2 configuration section.

For tension and compression a similar behavior may be assumed, provided that e.g. nothing else is stated in the approval.

High strength steel SAS according to approval Z-1.1-267:2016-04/2021-04 [72]:

To reach the yield point, a strain of 2.91 o / oo is required. This leads, particularly in the case of compression reinforcement, to the fact that the high steel strength can not be utilized.

Limits of the strain distribution in the ULS according to Figure 6.1:

Strain limit of the reinforcing steel	ε_{ud}
Compression limit of the concrete	ε_{cu2} *1)
Compression limit of the concrete with pure normal force	ε_{c2} *2)

*1): According to 6.1. (5) the compression in the center of the plate of articulated sections shall be limited to ε_{cu2} according to Tab. 3.1. This is implemented with the exception of annulus-, rectangular hollow- and polygonal cross sections.

*2): NA-D:

At low eccentricities ed / h < 0.1, $\varepsilon c2$ can be assumed to be 2.2 ‰.
This is implemented with the exception of annulus-, rectangular hollow- and polygonal cross sections.
For these cross-sections, the calculation is done always with $\varepsilon c2$ according to Tab.9, 10.

Minimum moment: According to 6.1 (4), M> N · max (2 cm, h / 30)
NA-D:
Not required in a second order analysis.

Design for a given reinforcement ratio

This function is particularly suitable for the design calculation when compressive force with low eccentricity applies. It can also be used universally, however, with multiaxial loading and circular cross sections, for instance. The breaking state is assessed by iterative calculation with a given reinforcement layout (biaxial loading) and/or a given ratio of tensile and compression reinforcement (uniaxial loading).

You can reduce the required steel quantity by selecting a particular reinforcement ratio or layout.

Minimum reinforcement

Where compression members (ed/h < 3.5) are concerned, the software checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design of T-beams, rectangular and layered cross sections, the software checks in addition whether the required minimum reinforcement for components under bending will become decisive.

For the design types biaxial design of rectangular and circular cross sections, the minimum reinforcement is currently not considered.

You can optionally disable the consideration of both minimum reinforcements in the section → B2 design configuration.

EN 1992-1-1

NA-D:	Tables for uniaxial loading in / 46 / (fck <= 50 N/mm^2)
	Circular and rectangular cross sections with d1/h = 0.05 ...0.20
NA-A:	Tables for uniaxial loading in / 48 / (fck <= 50 N/mm^2)
	Circular and rectangular cross sections with d1/h = 0.05 ...0.20
NA-GB:	Tables for uniaxial loading in / 50 / (fck <= 50 , fck = 90 N/mm^2)
	Circular and rectangular cross sections with d1/h = 0.05 ...0.20
NA-I:	Exemplary table for uniaxial loading in /58/ (fck=30 N/mm^2) Rectangular cross section with d1/h = 0.1
NA-PL	Exemplary tables for uniaxial loading in /64/ (fck<=50 N/mm^2) Rectangular cross section

Dimension-dependent design (kd method)

The method is used for the design of cross sections under uniaxial loading and is the preferable method for bending and longitudinal force with high eccentricity.

$k_d = \frac{d[cm]}{\sqrt{\frac{M_s[kNm]}{b[m]}}}$ is the measure of the effect of the cross section loading.

In the first place, the layout of a tensile reinforcement is assumed. The resisting moment for a strain state is calculated via the balance of the moments in regard to the reinforcement layer. The full utilization of the reinforcement produces the strain state with the maximum moment with the compressive limit strain of the concrete on the pressure side and the yield strain at the level of the reinforcement layer. If the applied internal moment is smaller than the limit moment, the breaking state is determined by iterative balancing of the moments and the axial forces. If the applied internal moment is greater than the limit moment, the strain state described above is assumed. The differential moment is balanced with compression reinforcement.

If compressive strains do not occur, the design is performed according to the lever principle.

In linear elastic calculations of continuous beams, the compression zone height should be limited if no constructive measures are undertaken. Compliance with this criterion is achieved by modifying accordingly the limit steel strain that requires the calculation of compression reinforcement.

Minimum reinforcement

Where compression members (ed/h < 3.5) are concerned, the software checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design of T-beams, rectangular and layered cross sections, the application checks in addition whether the required minimum reinforcement for components under bending will become decisive.

You can optionally disable the consideration of both minimum reinforcements in the section
→ B2 design configuration.

Specialities in the analyses on continuous beams without redistribution of the internal forces

The criterion for the calculation of a compressive reinforcement is whether the related compression zone height is exceeded. The compression zone height is calculated in accordance with 5.5 (4) with $\delta = 1.0$ (no redistribution).

$\frac{x}{d} = \frac{(\delta - k1)}{k2}$ or $\frac{x}{d} = \frac{(\delta - k3)}{k4}$ for $f_{ck} > 50\ N/mm^2$

	K1	K2	x/d	K3	K4	x/d (C90)
NA-EN	0.44	k4 = 1.25 (0.6 + 0.0014 / εcu2)	0.448	0.54	k4 = 1.25 (0.6 + 0.0014 / εcu2)	0.33
NA-D	0.64	0.8	0.45	0.72	0.8	0.35 *a)
NA-GB	0.4	k4 = (0.6 + 0.0014/ εcu2)	0.6	0.4	k4 = (0.6 + 0.0014/ εcu2)	0.53
NA-A	= EN	= EN	= EN	= EN	= EN	= EN
NA-I	= EN	= EN	= EN	= EN	= EN	= EN
NA-PL	= EN	= EN	= EN	= EN	= EN	= EN

NA-D *a): applies also to lightweight concrete

Minimum reinforcement for components exposed to bending

The minimum value of a longitudinal reinforcement exposed to tensile stress in accordance with 9.2.1.1 is a NDP.

	Asmin
EN	$= 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d > 0.0013 \cdot b_t \cdot d$
NA-D	$= \frac{M_{cr}}{(f_{yk} \cdot z)} + N) / f_{yk}$ with $M_{cr} = (f_{ctm} + \frac{N}{A_c}) \cdot W_c$ and $z = 0.9 \cdot d$ see /14/
NA-GB	= EN
NA-A	= EN
NA-I	= EN
NA-PL	= EN

Minimum reinforcement for compression members

In accordance with DIN 992-1-1/NA (NCI to 1.5.2.) compression members are cross sections under compression with a related eccentricity of ed/h <= 3.5. in the ultimate limit state. If biaxial loading applies, the criterion must be met in one of the two directions at least.

As,min	Columns	Walls
NDP As,min	Columns (9.5.2(2))	Walls (9.6.2(1))
EN	$= 0.10 \cdot \frac{N_{Ed}}{f_{yd}} > 0.002 \cdot Ac$	$= 0.002 \cdot Ac$
NA-D	$= \frac{0.15 \cdot N_{Ed}}{f_{yd}}$	$= 0.15 \cdot \frac{N_{Ed}}{f_{yd}}$ $0.003 \cdot Ac > As > 0.0015 \cdot Ac$
NA-GB	= EN	= EN
NA-A	$= 0.13 \cdot \frac{N_{Ed}}{f_{yd}} > 0.0026 \cdot Ac$	= EN
NA-PL	= EN	= EN

Lever principle

If the resulting longitudinal tensile force lies in the area of the reinforcement layers, no concrete compression zone is produced. To simplify the calculation, it is assumed that the reinforcement reaches the yield limit on bottom and on top. The size of the reinforcement then simply depends on the reinforcement spacing referenced to the centre of gravity of the cross section and the eccentricity of the resulting force and can be calculated according to the lever principle (DafStb H.220 1.2.8).

See in addition → Calculation of the effective stiffness.

Calculation of the effective stiffness

The state of strain in which the external and internal forces are in balance is sought after.

The calculation is based on three non-linear equations with three border strains as unknowns. They are resolved by iteration with the help of the Newton method.

The effective stiffness in combination with bending is consequently determined by the strains. The following equations apply
$EI_{y,eff} = M_y \cdot H / (\varepsilon 1 - \varepsilon 3)$ and

$EI_{z,eff} = M_z \cdot B / (\varepsilon 1 - \varepsilon 2)$.

H,B:	dimensions of the enclosing rectangle of the cross section
ε1:	Strain with maximum compression
ε2:	Strain in the adjacent corner in x-direction
ε3:	Strain in the adjacent corner in y-direction

Note concerning polygonal cross sections:

With general cross sections, uniaxial loading can also produce curvatures in the direction where the moment is equal to zero.

Therefore, you should take the curvatures instead of the effective stiffness into account in deformation calculations.

External and internal forces

You can optionally select whether the effective stiffness should be calculated in the serviceability limit state (SLS) or the ultimate limit state (ULS), → see Design configuration).

The resulting internal forces are determined by the internal action curves for concrete and steel.

EN 1992-1-1, ultimate limit state

Internal action curve of steel	Bilinear internal action curve as per figure 3.8 with the design values f_{yd} (yield limit) and $f_{td}(\varepsilon_{ud})$.
	Additional option: "Mean values of material parameters": $f_y = f_{yk}$ and $f_t(\varepsilon_{uk}) = f_y \cdot k$ (ε_{uk}, k as per Annex C) NA-D: Figure 3.8.1, NCI to 5.7 $f_y = 1{,}1 \cdot f_{yk}$ and $f_t(\varepsilon_{uk}) = fy \cdot k$ (ε_{uk}, k as per Annex C)

Internal action curve of concrete

If the stress-strain curve is enabled for the calculation of the internal forces (→ see B2 configuration), the internal action line of concrete as per figure 3.2 and 5.8.6 (3) applies with $f_c = f_{cd}$ and $k = E_{cm} / \gamma_{cE} \cdot \varepsilon_{c1} / f_c$, (E_{cm}, ε_{c1} and ε_{c1u} as per table 3.1 or table 11.3.1. γ_{cE} is a NDP). If it is not enabled, the parabola rectangle diagram in accordance with fig. 3.3 and the parameters as per table 3.1 or 11.3.1 apply.

	fc	γ_{cE}
EN	fcd	1.2
NA-D	fcm/γ_c	1,5
NA-GB	=EN	= EN
NA-A	=EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

Additional option "mean values of material parameters"
NA-D: 5.7 (6) et seq., supplementing NCCI

$f_c = 0.85 \cdot \alpha_{cc} \cdot f_{ck}$

$k = E_{cm} \cdot \varepsilon_{c1} / f_c$ (E_{cm}, ε_{c1} and ε_{c1u} as per table 3.1 or table. 11.3.1).

Other NAs as NA-D

EN 1992-1-1, serviceability limit state

Intern. action curve steel — Bilinear stress-strain curve, material coefficients are set to 1.0

Intern. action curve concrete — Linear internal action curve with E_{cm}

Internal forces — In the serviceability limit state SLS, the internal design forces of the ultimate limit state ULS are divided by a factor defined in the configuration or the internal forces of the quasi-permanent load combination are used → see B2 configuration.

Creep and shrinkage

If creep and shrinkage are enabled in the →B2 configuration , they are considered in the stiffness calculation as follows:

Creep: If the stress-strain curve of the concrete is non-linear (normally in the ULS), strain is modified in the calculation of the internal forces as per 5.8.6 (4)
$\varepsilon = \varepsilon/(1+\varphi)$ with $\varphi = \varphi(t0,\infty)$ as per Annex B

In order to take a diminished creep coefficient φeff as per 5.8.4. into consideration, the user must enter it manually
→ see B2 Environmental conditions/creep coefficient.

With a linear stress-strain curve, the software reduces the modulus of elasticity of the concrete as per eq. 7.20 with
$E_{ceff} = E_{cm}/(1+\varphi)$ in the calculation of curvatures in state I.

Shrinkage in state I:

Shrinkage is considered via an additional curvature

$1/r_S = \gamma_{cs} \cdot E_s/E_{ceff} \cdot S/I$ (equation 7.21)

ε_{cs}: shrinkage strain as per 3.1.4 and Annex B

S: static moment of the reinforcement relative to the centroid axis (state I) or the neutral axis (state II)

I: moment of inertia of the cross section (state I)

Shrinkage in state II:

According to /24/ p. 18, creep is taken into account via a negative compressive pre-strain of εcs in the calculation of the internal steel forces.

Tension stiffening

If the corresponding option is activated in the → B2 configuration, tension stiffening or the participation of the concrete between the cracks is considered by modifying the internal action curve of the reinforcing steel (cf. /14/ p. 35). Depending on the relationship between the steel strain under load in state II and the steel strain under internal crack forces, the steel strain is reduced due to tension stiffening acc. to /14/ figure H.8-3 to ε_{sm}.

Component stiffness : Only with the cross section types rectangle uniaxial, T-beams and layered cross section.

In accordance with equation 7.18, the distribution coefficient ζ provides for a weighting between

the curvatures in state II $1/r_{II} = (\varepsilon_2 - \varepsilon_1) / h)$ and

the curvatures in state I $1/r_I = M / (Ii \cdot E_{ceff}) + 1/r_S$

to an average curvature $1/r_m = 1/r_{II} \cdot \zeta + (1-\zeta) \cdot 1/r_I)$

$\zeta = 1 - ß \cdot (\sigma s/\sigma sr)^2$ equation 7.19

σsr: steel strain in state II exposed to internal crack forces calculated with $f_{ctk0.05}$ (default) or f_{ctm} (option),
→ see B2 design configuration.

σs: steel strain in state II under the load for which the stiffness is calculated (default) or in the infrequent load combination (option),
→ see B2 design configuration

Short-term loading: ß = 1.0 (ULS)

Long-term loading: ß = 0.5 (SLS)

$EIeff = M_y/(1/r_m)$

Cross-sectional stiffness: The effective stiffness is determined by the curvatures in state II using the factor
$k\zeta = (\varepsilon_{sm}-\varepsilon_{c2}) / (\varepsilon_{s2} - \varepsilon_{c2})$ to obtain
$E_{Ieff} = M / (k\zeta \cdot 1/r_{II})$ (cf. /22/ p. 303)

Shear design

Shear force

The analysis of the shear resistance is based on a truss model with compressive concrete struts and steel ties (stirrups). The minimum stirrup requirements result from the flattest possible strut inclination.

A flatter inclination reduces the bearing capacity of the struts, however, and increases in addition the forces in the tension chord. The result is an increased offset dimension.

Shear design for vertical shear reinforcement (stirrups):

VEd design value of the shear force (ULS)

VRd,c The shear resistance without reinforcement for the cracked state results from equation 6.2 or 11.6.2 for lightweight concrete

$VRd,c = CRdc \cdot \eta 1 \cdot k \cdot (100 \cdot \rho l \cdot fck)^{1/3} + k1 \cdot \sigma cp) \cdot bw \cdot d >= VRdc$ (eq. 6.2b)

CRdc: calibration factor acc. to 6.2.2: (1) (NDP)

K1: empirical strain coefficient

NDP	k1:	CRdc
EN	0.15	$0.18/\gamma c$ standard concrete $0.15/\gamma c$ lightweight concrete
NA-D	0.12	$0.15/\gamma c$
NA-GB	0.15,	$0.18/\gamma c$, > C50 test or as C50
NA-A	= EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

$\eta 1$ correction factor for lightweight concrete

K $=1+\sqrt{(200/d)} <= 2$ [d in mm]

scaling factor, decreases when the effective height increases

ρl $=Asl/(bw \cdot d) < 0.02$

tensile reinforcement Asl that goes beyond the considered cross section with lbd+d

$\sigma cp= NEd/Ac < 0.2 \cdot fcd$

stress (negative compression)

bw: lowest cross section width within the effective height

Equation 6.2.b

VRd,c > (vmin+k1 · σcp) · bw · d

NDP	vmin standard concrete	vl,min lightweight concrete
EN	$0.035 \cdot k^{3/2} \cdot fck^{1/2}$	$0.028 \cdot k^{3/2} \cdot fck^{1/2}$
NA-D	$0.0520/\gamma c \cdot k^{3/2} \cdot fck^{1/2}$ (d < 600) $0.0375/\gamma c \cdot k^{3/2} \cdot fck^{1/2}$ (d > 800)	0
NA-GB	= EN	$0.028 \cdot k^{3/2} \cdot fck^{1/2}$
NA-A	= EN	
NA-I	= EN	$0.030 \cdot k^{3/2} \cdot fck^{1/2}$
NA-PL	= EN	= EN

NA-GB: > C50 with fck= 50 N/mm2 or additional option "no reduction“

Optionally, the user can perform a calculation in the uncracked state as per equation 6.4 (see B2 configuration), if the concrete border strain is smaller than fctk 0.05/γc (NA-D: fctd).

NA-D: does not apply to pre-stressed element ceilings

Alternative: applies to single-span systems of pre-stressed concrete

$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \cdot \sigma_{cp} \cdot f_{ctd}}$$

I: moment of inertia

S: static moment in the decisive section

b_w: width in the decisive section

σ_{cp}: longitudinal stress in the decisive section

a_l: coefficient for pre-tensioning in the area of the transmission length, otherwise always 1.0

f_{ctd}: arithmetical value of the tensile strength of the concrete

$f_{ctd} = \alpha_{ct} \cdot f_{ctk}\ 0.05 / \gamma_c$

γ_c: partial safety factor (see Bases of design)

f_{ctk} 0.05: lower characteristic value of the tensile strength of the concrete

NDP	α_{ct} standard concrete as per 3.1.6	α_{ct} standard concrete as per 11.3.5
EN	1.0	0.85
NA-D	0.85	0.85
NA-GB	= EN	= EN
NA-A	= EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

When using equation 6.4 make sure that the decisive section is not in the centre of gravity of the cross section. It schould be determined by iteration if the cross section width varies or the longitudinal tension is inconstant. This means that VRdc also depends from the entered longitudinal force (minimum is decisive) and the entered bending moment (maximum is decisive).

Components with required shear reinforcement

Cot Θ The goal of the design is to minimize shear reinforcement, i.e. the flattest possible strut inclination angle (max Cot Θ) is sought after, at which the bearing capacity of the strut is still ensured.

If loading by torsion applies simultaneously, this bearing capacity can become decisive for the strut inclination angle to be selected.

NDP	Max Cot Θ	Min Cot Θ	Comment
EN	2.5	1.0	Determination of Θ based on VRd,max criterion
NA-D	3.0 standard concrete 2.0 lightweight concrete	0.58	Take additional crack fraction criterion into account
NA-GB	= EN 1.25 with external tension	= EN	= EN
NA-A	1.6 in general 2.5 with overpressure on cross section	= EN	= EN
NA-I	= EN	= EN	= EN
NA-PL	2.0	= EN	= EN

NA-D:

Cot Θ <= $(1.2 - 1.4 \cdot \sigma cd/fcd) \;/\; (1\text{-}VRd,cc/VEd)$ eq. 6.7aDE

VRd,cc: Crack friction force

$VRd,cc = ßct \cdot 0.1 \cdot fck^{1/3} \cdot (1 - 1.2 \cdot \sigma cd/fcd) \cdot bw \cdot z$ eq. 6.7.bDE

You can optionally set the strut inclination angle by default (→ B2 design options) to analyze additional sections with the strut inclination angle relevant at the decisive cross section, for instance. This angle must not be flatter than the required one.

z lever arm of the assumed framework model according to the bending design (if unknown, assumption of 0.9 · d, or of 0.55 · d with circular cross sections).

NA-D: limitation z < d - 2 · cv,l (here cv,l = nomc of the longitudinal reinforcement in the compression zone, acc. to /26/, a limitation of z < d – cv,l - 3cm applies to cv,l > 3cm).

You can also set a user-defined lever arm by default (→ B2 design results).

aswV calculated shear reinforcement acc. to equation 6.8

The selection of the strut inclination angle also in line with the criterion for compliance with VRdmax proves equation 6.12.

The software checks whether a minimum shear reinforcement acc. to 9.2.2 (5) for beams or 9.3.1.4 (NAD_D) for plates will become decisive. The reinforcement is calculated for an average web width (with circular cross sections bwS = Ac/Da).

With circular cross sections, an efficiency factor for round stirrups is calculated in accordance with /31/ that increases the required shear reinforcement. This factor takes into account that the applying shear force in normally not parallel to the resisting force of the stirrup. Depending on the considered section, the resisting force applies at a different angle to the perpendicular.

Min asw/s= $\rho \cdot bw \cdot \sin \alpha$

	ρ (beams) as per 9.2.2:	ρ (plates) as per 9.3.2:	Comment
EN	0.08 · √fck/fyk	0	
NA-D	0.16 · fctm/fyk	0 if VEd < VRdc Otherwise 0.6 *ρ	Junction area 4 < b/h < 5: Interpolation between 0 and the simple value (VEd < VRdc) or between 0.6 and the simple value (VEd > VRdc)
NA-GB	= EN	= EN	
NA-A	0.15 · fctm/fyk	= EN	
NA-I	= EN	= EN	Draft NA
NA-PL	= EN	= EN	

VRd,max The bearing capacity of the struts results acc. to 6.9 or equivalent and depends only on cot Θ. The following equation applies:

VRd,max= $bw \cdot z \cdot \alpha cw \cdot \nu 1 \cdot fcd \cdot \cot \Theta/(1+\cot^2 \Theta)$

NDP	ν1 acc. to 6.2.3	Comment
EN	ν1 = 0.6 · (1-fck/250) ν1 = 0.5 · (1-fck/250)	equation 6.6N equation 11.6.6N lightweight concrete
NA-D	ν1 = 0.75 * (1.1-fck/500) * η1	Standard concrete > C50 Lightweight concrete
NA-A	= EN	
NA-GB	ν1 = 0.6 · (1-fck/250) ν1 = 0.5 · (1-fck/250)	equation 6.6N equation 11.6.6N lightweight concrete
NA-I	ν1=0.5 [1] ν1 = 0.5 ·η1 (1-fck/250) [4]	Standard concrete Lightweight concrete
NA-PL	= EN	

All NAs: the increase by including only 80 % of the stirrup bearing capacity in acc. with equation 6.10a and 6.10b is not considered.

For reinforced concrete: αcw = 1.0

NA-GB: > C50 with fck= 50 N/mm2 or additional option "no reduction“

(see B2 configuration)

PD 6687:2006 chapter 2.3 allows the caluclation of fcd with αcc=1.0.

(Option "Increased fcd as per PD 6687:2006“ see B2 configuration)

The maximum of VRd,max results for a strut inclination angle of 45°.

If VRd,max is smaller than the design value of the shear force, you should increase the cross section or the concrete class.

bw The width bw corresponds to the web width b0 for T-beams and to the lowest width in the cross section for layered cross sections. Where circular cross sections are concerned, bw corresponds to the lowest width between the resultant compression force and the resultant tension force. If the position of the resultant force is unknown (moment and axial force are equal to zero) a safe distance of the resultant compression force of Da/40 is assumed in the calculation.

sl,max maximum stirrup spacing as per 9.2.2 (6)

	sl,max (NDP acc. to 9.2.2 (6)
EN	$0.75 \cdot d \cdot (1+\cot\alpha)$
NA-D	distinguished according to shear force utilization with a VRdmax (Θ = 40°)
NA-GB	= EN2
NA-A	$0.75 \cdot d \cdot (1+\cot\alpha) <= 250$ mm
NA-I	= EN
NA-PL	= EN

NA-D:

VEd < 0.3 · VRdmax sMax = 0.7 · h beam: < 30 cm (> C50/60: < 20 cm)

VEd < 0.6 · VRdmax sMax = 0.5 · h beam: < 30 cm (> C50/60: < 20 cm)

VEd > 0.6 · VRdmax sMax = 0.25 ·h beam: < 20 cm

VRdmax may be assumed with θ= 40 degrees according to /14/ p. 212

Biaxial shear force for rectangular cross sections

In accordance with the method described in reference /39/, the verification is reduced to the uniaxial scenario with the help of adjusting factors for the load-bearing capacity of the struts and the stirrups.

Boundary case 1 is uniaxial loading with $\alpha_v = 0$, boundary case 2 is biaxial loading with an accurately diagonal load application of the resultant, i.e. $\alpha_v = 1$.

In accordance with reference /39/, the force in the stirrup for case 2 is as follows:

$2 \cdot V_z = 2 \cdot \frac{V_{Ed}}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}}$, which means it is $\frac{2}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}}$ times greater than in case 1.

The highest loading on the compressive concrete strut results in case 2 for the load transfer point from the strut to the tension chord, where the width b_{eff} is reduced to 0.6 · b according to the conservative estimation prescribed in reference /39/. When assuming the same lever arm in both cases, the compressive strut loading resulting in case 2 is b/b_{eff} times higher than in case 1.

Between these two cases, interpolation is performed in accordance with the existing inclination α_v with the help of the following relations:

V_{Ed}: resulting shear force $\sqrt{V_{Edy}^2 + V_{Edz}^2}$

α_v: related shear force inclination $\frac{|V_{Edy}| \cdot h}{|V_{Edz}| \cdot b}$

h: side length in the z-direction

b: side length in the y-direction

if 0 <= α_v <= 1,

then bearing strength verification with $b_w = b$,

otherwise $\alpha_v = 1/\alpha_v$ and bearing strength verification with $b_w = h$,

$$V_{Rd,sy} = V_{Ed} = \frac{A_{sw}}{sw} \cdot f_{yd} \cdot z \cdot \cot\theta \cdot \frac{1}{k_{asw}}$$

Interpolation factor for shear reinforcement

$$k_{asw} = 1 + \left(\frac{2}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}} - 1 \right) \cdot \alpha_v^{1/2}$$

$$V_{Rd,max} = b \cdot z \cdot \alpha_c \cdot \frac{f_{cd}}{\cot\theta + 1/\cot\theta} \cdot k_{vmax}$$

Interpolation factor for compressive strut resistance

$$k_{vmax} = \frac{1}{1 + \left(\frac{b}{b \cdot 0{,}6} - 1 \right) \cdot \alpha_v^{1/2}}$$

z lever arm of the inner forces, i.e. the distance between the tension resultant and the compression resultant, as resulting from the bending design.

If the lever arm is unknown, interpolation is performed between z = 0.9 · (h-d1) for $\alpha_v = 0$ and z = 0.9 · (h-d1+b-b1)/2 for $\alpha_v = 1.0$ in relation to the existing α_v.

NA_D: z < d - 2 · nomc

This limitation shall ensure that the distance of the compression resultant to the compressive edge is not smaller than 2 · nomc.
Consequently, d refers to the distance of the tension resultant for the compressive edge in the direction of the lever arm.

V_{Rdc} is calculated by approximation with $b_w = 0.6 \cdot b_w$ (case 1) and d = z.

Cast-in-place complement

For cross sections with cast-in-place complement, the bearing capacity of the cast-in-place joint is to be verified vEdi < vRdi equation 6.23

vEdi shear force to be transmitted per length unit in the joint

vEdi = ß · VEd / (z · bi)equation 6.24

VEd: design value of the shear force

z: lever arm of the internal forces, see shear resistance verification

NAD_D: if VRd,c > VEd, the lever arm limitation with cv can be dispensed with.

ß: ratio of axial force in the cast-in-place concrete to total compression force (assumption 1.0)

vRdi design value of the shear force resistance of the joint

vRdi = c ·fctd + $\mu \cdot \sigma$n + ρ · fyd · ($\mu \cdot \sin\alpha + \cos\alpha$) < 0.5 · ν · fcd
(equation 6.25, lightweight concrete with dctd = flctd and ν= νl and fcd= flcd)

σn axial stress perpendicular to the joint with σND = nEd/bi < 0.6 · fcd

nEd: design value (compression: lower, tension: upper) of the axial force perpendicular to the joint per length unit, negative compression.

bi: effective joint width, reduced total width due to prefabricated formwork, if applicable.

c roughness coefficient according to surface quality

Very smooth	Smooth	Rough	Interlocked
0.1	0.20	0.40	0.50

μ friction coefficient according to surface quality as per table 13

Very smooth	Smooth	Rough	Interlocked
0.5	0.6	0.7	0.9

ν strength reducing coefficient as per 6.2.2 (6)

ν	Very smooth	Smooth	Rough	Interlocked
EN Standard concrete Lightweight concrete	 0.6 · (1-fck/250) 0.5 · (1-fck/250)	 0.6 · (1-fck/250) 0.5 · (1-fck/250)	 0.6 · (1-fck/250) 0.5 · (1-fck/250)	 0.6 · (1-fck/250) 0.5 · (1-fck/250)
NA-D (NCCI) Standard concrete > C50 Lightweight concrete	 0.0 0.0 * η1	 0.2 * (1.1-fck/500) * η1	 0.5 * (1.1-fck/500) * η1	 0.7 * (1.1-fck/500) * η1

NA-D:
$vRdi = c \cdot fctd + \mu \cdot \sigma n + \rho \cdot fyd \cdot (1.2 \cdot \mu \cdot \sin\alpha + \cos\alpha) < 0.5 \cdot \nu \cdot fcd$
(eq. 6.25 + NCI or eq. 11.6.25 for lightweight concrete, with fctd= flctd and $\nu = \nu l$ and fcd= flcd)

very smooth with c= 0

ρ shear reinforcement ratio of the joint

$\rho = Asw / Ai = asw / bi$

asw required stirrup reinforcement crossing the joint, hence vRdi = vEdi
$vrdi0 = c \cdot fctd + \mu \cdot \sigma n$ bearing capacity without joint reinforcement
$asw = bi \cdot (vEdi - vRdi0) / (fyd \cdot k \cdot \mu \cdot \sin\alpha + \cos\alpha)$
NA-A:
$asw > Min = \rho_{min} * b$
Plates: $\rho_{min} = 0.12 \cdot f_{ctm} / f_{yk} > 0.0005$
Beams: $\rho_{min} = 0.20 \cdot f_{ctm} / f_{yk} > 0.001$
The verification of the anchorage required by the National Annex is not implemented currently. A successful result is presumed, however, because asw is calculated with fyd without reduction.

Torsion

Torsion design is done with the help of an equivalent hollow cross section. With structured cross sections, only the web cross section is used in the approach by approximation.

tef,i: effective wall thickness

tef,I = A / U

< 2 · d1 double spacing of reinforcement

< ba real wall thickness with hollow cross sections

The requirement to verify explicitly torsional resistance instead of the minimum reinforcement results from the interaction equation 6.31 that is different in NA-D.

NA-A, NA-GB:

TEd/TRdc + VEd/VRd,c < 1 equation 6.31

TEd: design value of the torsional moment

TRdc: resisting torsion moment only depending on the tensile strength of the concrete

TRdc= fctd · t · 2 · Ak as per /55/ p. 6-13

Wt: section modulus as per /46/ p. 309

NA-D:

TEd < VEd · bw/4.5 equation 6.31aDE

VEd · (1+ (4.5 · Ted) / (VEd · bw)) <= VRdct equation 6.31bDE

Cot Θ The goal of the design is to minimize shear reinforcement, i.e. the flattest possible strut inclination angle (max Cot Θ) is sought after, at which the bearing capacity of the strut is still ensured.

This calculation does not automatically produce the reinforcement minimum because the portion of the longitudinal torsion reinforcement increases considerably with flatter struts.

If shear loading applies simultaneously, the interaction of shear force and torsion might become decisive for the design.

To simplify the calculation, you can base the torsion analysis exclusively on the assumption Cot Θ = 1.0 (45 degrees) (see Design configuration).

NA-D:

Calculation of the strut inclination angle acc. to /51/, p. 173 ff

Cot Θ <= (1.2 - 1.4 · σcd/fcd) /(1-VRd,cc/VEd, T+V) acc. to equation 6.7.aDE

VEd, T+V: resultant loading

VEd,T+V = VEd,T + VEd,V · teff,I / bw

VEd,V: loading by shear force

VEd,T: loading by torsion
VEd,T = Ted · zi / (2 · A)

VRd,cc: crack friction force acc. to eq. 6.7.bDE

VRd,cc = ßct · 0.1 · $fck^{1/3}$ · (1 - 1.2 · σcd/fcd) · tef,i · z

TRd,max design value of the resisting torsional moment acc. to equation 6.30 or equivalent depending only on cot Θ. The following equation applies:

$Trd,max= 2 \cdot \nu \cdot \alpha cw \cdot fcd \cdot Ak \cdot tef,I \cdot \cot\theta\ (1 + \cot^2\theta)$

Ak: area enclosed by the wall centre lines

NDP	ν (6.2.2. (6))	Comment
EN	 ν = 0.6 · (1-fck/250) ν = 0.5 · η1 · (1-fck/250)	Analogously to shear force Standard concrete Lightweight concrete
NA-D (NCCI)	 ν = 0.525 * (1.1-fck/500) * η1	Reduced in comparison to shear force Standard concrete > C50 Lightweight concrete
NA-A	= EN	
NA-GB	= EN	
NA-D (NCCI)	 ν=0.5 ν= 0.5·η1· (1-fck/250) [4] p. 63	Analogously to shear force Standard concrete Lightweight concrete
NA-PL	= EN	

αcw: coefficient analogous to VRd,max

The maximum for TRd,max results for a strut inclination angle of 45 degrees. If TRd,max is smaller than the design value of the torsional moment, you should increase the cross section or select a higher concrete class.

aswT the required stirrup reinforcement due to torsion results from

$aswT^* = TEd/(2 \cdot Ak \cdot fyd \cdot \cot\theta)$ /46/ p. 283

The minimum shear reinforcement becomes decisive if aswV+ aswT < aswMin is true.

The required shear reinforcement aswT is specified in relation to the total cross-section. Since aswT is determined by the program only for one wall of the hollow cross-section, the output is therefore double the value (aswT= = 2*aswT*). The background is the simpler superposition with a shear force stress.

See Zehetmayer,Zilch: "Bemessung im konstruktiven Betonbau", Springerverlag, Berlin 2010, 2nd edition, p. 308

AsL additional longitudinal reinforcement due to torsion

$Asl = TEd \cdot \cot\theta \cdot Uk/(2 \cdot Ak \cdot fyd)$ eq. 6.28

Uk: circumference of area Ak

With combined shear force and torsional loading, the following interaction condition must be complied with:

$TEd/TRd,max + VEd/VRd,max < 1$ equation 6.29

NA-D and NA-A:

For compact cross section applies

$(TEd/TRd,max)^2 + (VEd/VRd,max)^2 < 1$ NA-D: eq. NA.6.29.1/NA-A: eq. (9)

The stirrup cross section results from asw(V+T)= aswV+ aswT.

Shear design for prefabricated floors with lattice girders:

The verification for DIN EN 1992-1-1/NA can be performed on the bases of manufacturer-specific approvals (e.g. ref. /67/.../72/).

Lattice girders consist of a compression chord, a tension chord and struts.

The struts can either have the shape of isosceles triangles
(inclination angle of 45° <= α < 90° e.g. ref. /67/, /69/, /71/, referred to as "isosceles triangle" in the following structural system) or consist of a vertical post and a diagonal strut
(inclination angle of 45° <= α1 < 90° e.g. ref. /68/, /70/, /72/., referred to as "post/diagonal strut" in the following structural system).

The following limitations apply:

- Permissible only for plates (w/h $\geq$ 5 or option „Like plate")
- Minimum thickness of 4 cm
- Concrete grades < C50/60 or < LC50/55 with a raw density class of D1.2
- "Isosceles triangle" system only permissible for mainly steady live loads

<u>Design for shear force resistance:</u>

VRdc — In derogation of the design standard, longitudinal compression stress must not be taken into account.

Cot Θ — in derogation of the design standard, the lower limit is Cot Θ >= 1.0.

in derogation of the design standard, longitudinal compression stress must not be taken into account.

aswQ — the required shear reinforcement is calculated using eq. 6.13 in accordance with the inclination angle α of the struts. For the system post/diagonal strut, it is assumed that the diagonal strut (α=α1) and the post (α=90 degrees) bear 50 % of the load each.

If the struts are made of smooth reinforcing steel B 500 A+G, a fyd-value of merely fyd= 365 N/mm² may be taken into account.

VRd,max — is calculated using eq. 6.14 in accordance with the inclination angle α of the struts. In derogation of the relevant standard, the following applies in accordance with eq. 6.14: VRd,max,GT= 1/3* VRd,max.

For the post/diagonal strut system, the verification is based on an interaction equation Σ(VRdsy, α_i / VRdmax, α_i) <= 1,0 due to the different inclinations of the struts. (See ref. /66/ eq. H.6-7)

VRdsy, α_i: bearing capacity portion of the strut with the angle α_i

VRdmax,α_i: bearing capacity of the compressive strut with assumption of a strut inclination angle α_i

If the verification is not successful, the cross section or the concrete class should be increased.

sl,max — maximum distance of the diagonal strut in the supporting direction as per ref. /67/ to /72/

smax= (cot θ + cot α) · z <= 20 cm

<u>Shear force transmission in the joint:</u>

In derogation of the verification method described in chapter 6.2.5, the limitation of vRdi,max for standard concrete and lightweight concrete in accordance with the manufacturer-specific approvals (/67/ - /72/) applies in addition.
If the verification of the shear force resistance reveals that VEd < VRdc, the lever arm limit
z < max.(d- 2* cvl, d- cvl- 3 cm) is not taken into account in the calculation of vEd.
(See ref. /66/ concerning 6.2.5 (1))

Serviceability verifications

Crack width verification in accordance with EN 1992-1-1

Based on the crack formula equation 7.8 $wk = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$

the maximum limit diameter still in compliance with the permissible crack width is calculated for an external loading that depends on the decisive combination of actions and a pre-selected reinforcement.

Decisive combinations of actions and permissible crack width as per table 7.1 (NDP)

The considered NAs all require the verification of a permissible crack width of 0.3 mm for reinforced concrete components of exposure class XC2 and higher.

The verification for XC1 is based on a crack width of 0.4 mm for aesthetical reasons (exception GB: 0.3 mm)

Under normal conditions, the quasi-permanent load combination (Qk) is the decisive one.

Considerably different requirements apply in Italy and the Netherlands.

Requirements referring to reinforced concrete components as per table 7.1.

	X0, XC1	XC2/XC4	XS1-3, XD1-3	Comment
EN	0.4 mm + Qk	0.3 mm + Qk	0.3 mm + Qk	Tab. 7.1N
NA-D	= EN	= EN	= EN	Tab. 7.1DE
NA-GB	0.3 mm + Qk	= EN	= EN	
NA-A	= EN	= EN	= EN	
NA-I	AO 0.3 mm + Qk 0.4 mm + Hk	AA 0.2 mm + Qk 0.3 mm + Hk	AM 0.2 mm + Qk 0.2 mm + Hk	A0,AO,AA,AM as per NTC tab. 4.1. III
NA-PL	= EN	= EN	= EN	

Due to the fact that the tensioning steel is highly susceptible to corrosion, pre-stressed concrete components have to comply with higher requirements in regard to the load combinations (infrequent (Sk), frequent (Hk)) to be verified and the permissible crack width. In some cases, a verification of decompression (dec.) might be required.

The regulations differ in the various National Annexes.

Bonded pre-stressed concrete:

	X0, XC1	XC2/XC4	XS1-3, XD1-3	
EN	0.2 + Hk	0.2+ Hk Dec. Qk	Dec. Hk	Tab. 7.1N
NA-D	= EN	= EN	Bonded post-tensioned concrete: 0.2+ Hk and dec. Qk Bonded pre-tensioned concrete 0.2 + Sk and dec. Hk	Tab. 7.1DE
NA-GB	= EN	= EN	= EN	
NA-A	= EN	= EN	Bonded post-tensioned concrete: 0.2+ Hk and dec. Qk Bonded pre-tensioned concrete 0,2 + Sk and dec. Hk	
NA-I	AO 0.3 mm + Qk 0.4 mm + Hk	AA 0.2 + Hk dec.+ Qk	AM dec. + Qk Sigt + Sk	A0,AO,AA,AM as per NTC tab. 4.1. III
NA-PL	= EN	= EN	= EN	

The crack width results from the maximum crack spacing *srmax* and the average strain difference εsm - εcm of concrete and steel.

ε_{sm}- ε_{cm}: average strain difference between steel and concrete (equation 7.9)

$$\varepsilon_{sm}-\varepsilon_{cm}=\frac{\sigma_s-k_t\frac{f_{ct,eff}}{\rho_{p,eff}}\left(1+\alpha_e\rho_{p,eff}\right)}{E_s}\geq 0.6\frac{\sigma_s}{E_s}$$

k_t: 0.6 short-term action (not considered in the software)

0.4 long-term action

σs: steel strain in state II

calculation with $E_{ceff} = E_{cm}/(1 + \varphi\ (t=\infty))$

α_e = E_s / E_{ceff}

ρ_{eff}: reinforcement ratio in the effective tension zone

$\rho_{eff} = (A_s + A_p * \xi 1^2) / A_{ceff}$

A_s: reinforcing steel area included in A_{ceff}

A_p: tensioning steel area included in A_{ceff}

ξ: factor for the bond characteristics of tensioning steel

A_{ceff} : area of the effective tension zone

$A_{ceff} = h_{eff} \cdot b_{eff}$

h_{eff} 2.5 · D1 < (h-X0II)/2

X0II: compression zone height in state II:

if no reinforcement with spacing < heff

was defined, h_{eff} = (h-X0I)/2 applies

b_{eff} effective tension zone width for T-beams

NA-D:

as per /5/ p. 191 in accordance with the permissible relocation width of the tensile reinforcement

$b_{eff} <= \sum(0.5 \cdot b_{eff,i}(Z.I)) + bw <= bf$ (NCI zu 9.2.1.2 (2))

Input: see B2 dialog for Control of the crack width verification

Sr, max: maximum crack spacing:

$$s_{r,max} = k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p,eff}}$$

k_1: coefficient reinforcement bond quality

0.8 good bond quality

1.6 poor bond quality

k_2: coefficient of strain distribution

Bending: 0.5

Tension 1.0

Bending + tension $(\varepsilon 1 + \varepsilon 2) / (2 \cdot \varepsilon 1)$

c: concrete cover on longitudinal reinforcement

ϕ: average diameter of the tensile reinforcement

NDP	k_3	k_4
EN	3.4	0.425
NA-D	0	$1/(3,6 \cdot k_1 \cdot k_2) < \sigma_s \cdot \rho_{p,eff} / (3,6 \cdot k_1 \cdot k_2 \cdot f_{ct,eff})$
NA-GB	= EN	= EN
NA-A	0	$1/(3.6 \cdot k_1 \cdot k_2) < \phi \cdot \sigma s/(3.6 \cdot f_{ct,eff})$
NA-I	= EN	= EN
NA-PL	= EN	= EN

NA_D: For lattice girders with approval by the construction authorities, ref. /67/ .../72/ with smooth reinforcing steel in the chord, reduced bond stress can be taken into account.
In accordance with the bond stress for smooth bars specified by e.g. DIN 1045 /78 a factor of 1/0.388, which is on the safe side, results for the crack width.
This factor is also suitable for the calculation of the limit diameters specified in the tables of the approvals.

The limit diameter ϕ is obtained by rearranging the crack equation.

More favourable (larger) limit diameters than specified in table 7.2 may result because the simplifications the table is based on are dispensed with.

If the resultant limit diameter cannot be realized, you should increase the selected reinforcement.

For circular cross sections, $\rho_{eff} = A_s / A_{c,eff}$ is calculated for a circular ring with a thickness of h_{eff} because an evenly distributed reinforcement is assumed in accordance with reference /30/.
The expression $A_{c,eff} = \pi\ (D \cdot h_{eff} - h_{eff}^2)$ allows a more accurate determination.
The condition $A_{c,eff} <= A_c$ applies to circular ring cross sections in addition.

The results comply well with reference /30/ if the specified condition of n = 10 is satisfied by taking low creep factors into account. The results for $t = \infty$ are less favourable, however, because the creep factors are higher then.

Minimum reinforcement due to indirect action

The software application calculates a minimum reinforcement acc. to 7.3.2 for imposed bending on top and bottom if the corresponding option was enabled in the Control of the crack width verification dialog.

The minimum reinforcement for T-beams is calculated separately for the web and the flange, whereby the rectangle over the total cross section height is considered as the web and the remaining parts of the plate as the flange. You can take different bar diameters for flange and web into account.

$A_{s,min} \cdot \sigma s = kc \cdot k \cdot f_{ct,eff} \cdot A_{ct}$ (equation 7.1)

k — coefficient for non-linearly distributed internal stresses

1.0 (h <= 300 mm)... 0.65 (h >= 800 mm)

h: web height or flange width

NA-D: lower value of the partial cross section

if internal action applies, $k \cdot 0.8$

$f_{ct,eff}$ — tensile strength, f_{ctm} (t <= 28d)

NA-D: >= 2.9 N/mm^2 when t >= 28 d

k_c — coefficient for the stress distribution

$kc = 0.4 \cdot (1 - \sigma c / (k_1 \cdot f_{ct,eff} \cdot h/h'))$

σc: concrete stress (state I) under internal crack forces in the centre of gravity of the partial cross section

Flanges hollow box, T-cross sections, for internal crack forces completely under tension

$kc = 0.9 \cdot F_{cr} / (A_{ct} \cdot f_{ct,eff}) >= 0.5$

Fcr: tensile force in the flange under internal crack forces (state I)

σs: — Tab. 7.2N with Ds1, derivation see /54/ p. 7-6

$D_{s1} = D_s \cdot f_{ct0} / f_{ct,eff} \cdot 2 \cdot (h-d) / (k_c \cdot h_{cr})$

NA-D, NA-A:

As is calculated directly if $Fs = F_{cr} = k \cdot k_c \cdot f_{cteff} \cdot A_{ct}$.

$F_{cr} < F_{cre} = A_{ceff} \cdot f_{cteff}$

$$As = \sqrt{\frac{ds \cdot (1 - \beta t) \cdot Fs \cdot Fs}{3.6 \cdot Es \cdot wk \cdot fcteff}}$$

Otherwise

$$As = \sqrt{\frac{ds \cdot Fcre \cdot (Fs - \beta t \cdot Fcre)}{3.6 \cdot Es \cdot wk \cdot fcteff}}$$

Strain verification in accordance with EN 1992-1-1

Concrete, infrequent combination

$\sigma c < k1 \cdot fck$

The objective is to prevent the destruction of the concrete structure. Alternatively, you can increase the concrete cover or enclose the compression zone with reinforcement.

Concrete, quasi-permanent combination

$\sigma c < k2 \cdot fck$

When this limit value is exceeded, linear creep can no longer be assumed. If applicable, an increased creep coefficient according to equation 3.7 should be considered.

Reinforcing steel, infrequent combination

$\sigma s < k3 \cdot fyk$

Whereas the crack width verification for reinforced concrete is performed for the quasi-permanent combination, yielding of the reinforcement should also be prevented if the infrequent combination applies.

With indirect action: $\sigma s < k4 \cdot fyk$

	k1	k2	k3	k4	Comment
EN	0.6	0.45	0.8	1.0	k1: recommended with the exposure classes XD, XS or XF.
NA-D	= EN	= EN	= EN	= EN	k1: can be dispensed with where unpre-stressed components in typical building construction are concerned if the percentage of the redistribution is < 15 %.
NA-GB	= EN	= EN	= EN	= EN	
NA-A	= EN	= EN	= EN	= EN	
NA-I	= EN	= EN	= EN	= EN	k1: reduced by 20 % if h <= 50 mm
NA-PL	= EN	= EN	= EN	= EN	

Calculation of the existing stresses

In accordance with /11/, the steel stresses should be calculated with a reduced modulus of elasticity

Eceff = Ecm/(1+φ(t0, ∞)).

This calculation method takes the long-term behaviour of concrete into account. The concrete withdraws from its participation in load bearing by creep i.e. by redistribution to the reinforcing steel.

Acc. to /11/, this can often be neglected where compact cross sections are concerned. With T-beams, however, the resultant steel stresses increase by 5 % in comparison to a calculation that does not consider the creep coefficient. A corresponding note as in ENV 1992 1-1 Para. 4.4.1.3 (3) is however missing in EN 1992 1-1.

Correspondingly, early points in time are decisive for the calculation of the concrete stresses, i.e. φ = 0 in this case.

NA-A:

Reinforcing steel stresses with the accidental load combination:

Equation: $\varphi_{eff}(t0,\infty) = \varphi(t0,\infty) \cdot \frac{M_{qp,k}}{M_{E0,k}}$

$M_{qp,k}$: bending moment with the quasi-permanent load combination

$M_{E0,k}$: bending moment with the infrequent load combination

Reinforcing steel stresses with the infrequent load combination:

According to the NA, a calculation with $\varphi eff(t0,t)$ with t = start of usage is possible. This option is currently not implemented due to its insignificance.

Concrete stresses in the quasi-permanent load combination:

Unpre-stressed load bearing structures always with φ(t0, ∞).

This assumption is implemented as default in B2.

Accidental design situation fire

The design or the calculation of the stiffness for rectangular and circular cross sections with fire exposure on 1, 3 and 4 sides is implemented. (Note: B5 currently only 4-sides).

Fundamental considerations

The verification is performed in accordance with the requirements applying to a general calculation method. It includes a FEM-based temperature analysis with the parameters defined in the National Annexes (TA module is required) and a mechanical analysis to determine the internal forces with the help of the stress-strain curves of concrete and steel of EN 1992-1-1 and the determination of the balance with the external forces with consideration to thermal strain.

B2 application - reinforced concrete design
As the exact location and position of the steel is decisive for the result, the additional module "Polygonal design" B2-Poly should be available. The verifications under fire exposure are performed for the cross section types "rectangle with general point reinforcement" and "circle with general point reinforcement".

If the TA add-on module is not available, temperatures can be assessed by approximation with the help of the diagrams in EN 1992-1-2 Annex A. In this case, results may be non-compliant with the assumptions specified in some National Annexes, however.

Border conditions for the temperature analysis in the various National Annexes

	Component moisture %	Density ρ [kg/m^3]	Conductivity λ as per NA
EN (Annex A)	1.5	2300	λu
NA-D	3	2400	λo
NA-A	= EN	= EN	= EN High strength: λo
NA-GB	= EN	= EN	= EN High strength: λo
NA-PL	= EN	= EN	= EN

Note: *Component moisture and density are no NDPs. In Germany, these parameters do not comply with the assumptions stated by EN 1992-1-2 Annex A, however. See approximation method as per DIN EN 1992-1-2/NA Annex AA for instance.*

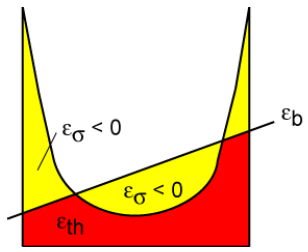
External forces

Forces of the combination for the accidental design situation fire should be used in accordance with EN 1990. In contrast to EN 1990, EN 1991-1-2 allows the use of a quasi-permanent value of ψ 2.1 · Qk,1 for the decisive variable action.
(NA-D: not allowed if wind is the leading action).

Internal forces

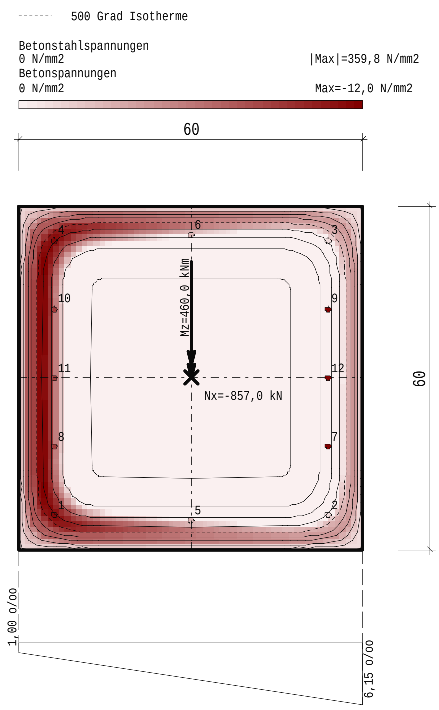
In order to calculate the internal forces acting on the concrete, the concrete cross section is divided into elements with an edge length of 1 cm each. The internal forces of the element result with the stress-strain curves corresponding to the average element temperatures acc. to /42/ figure 3.1 and table 3.1. Calcereous aggregates can be taken into account, if applicable. The thermal strain results according to figure 3.5. For high-strength concretes, modified stress strain curves as per table 6.1 N are used (NA-A: table 1):

The internal forces on the reinforcing steel depend on the temperatures in the reinforcement points acc. to /42/ figure 3.3 and table 3.2. The more favourable behaviour of hot-rolled steel can be taken into account in this connection, if applicable. According to /44/, steel of class X requires a proof by experimental testing and is therefore currently not supported. The thermal strain results according to /42/ figure 3.



The stress-generating strain ε_σ in an arbitrary point of the cross section results from the thermal strain ε_{th} depending on the temperature and the bending strain ε_b in this point. The equation $\varepsilon_\sigma = \varepsilon_b - \varepsilon_{th}$ applies.

A typical bearing behaviour results for the concrete, whereby a smaller outer ring due to the considerably diminished stress-strain curve at high temperatures and an inner area with $\varepsilon_\sigma > 0$ (tension) withdraw from their participation in the bearing of the loads.



The internal forces on the reinforcing steel react quite sensibly to the location of the reinforcement point, a minor change in position of 1 cm produces a measurable change in the steel strain.

The internal forces acting on the steel are calculated with consideration to the individual rebars. The effective stiffness results from the found strain state.

Design

The strain state (bending plane) at which the internal and external forces are in balance is sought after by iterative approximation.

The internal forces on the steel are first calculated for a reinforcement area still unknown whereby a uniform weighting of the entered reinforcement points is assumed.

The strain plane is varied between the defined breakage strains. The required reinforcement quantity results directly from the resultant strain state.

Calculation of the effective stiffness

→ See Calculation of the effective stiffness.

Validation examples

According to DIN EN 1991-1-2/NA, the software applications used for the general verification method should be validated with the help of the examples specified in Annex CC. Validation examples within the verification range of B2 are CC4.8 and CC4.9 - weakly and strongly reinforced beams.

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